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The Impact of Input and Output Decisions on Agricultural Production Risk

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This paper investigates the measurement of risk exposure in agriculture and its linkages with input and output decisions. We develop a conceptual analysis of risk under general risk preferences, including cumulative prospect theory. The approach is applied to a sample of U.S. farms from 1996 to 2011. In a multi-input, multi-output framework, the analysis documents the effects of management on production risk exposure and estimates the cost of risk under alternative frameworks. We find that variable inputs contribute to increasing risk, while livestock contributes to reducing risk. Nonfarm income reduces the cost of risk.

Key words: insurance, management, quantile, risk

Introduction

Risk is pervasive in agriculture. Unpredictable insect damages or weather shocks can have large adverse effects on farm production. Risk exposure has generated much interest in the economics of risk in agriculture (e.g., Harwood et al., 1999; Just and Pope, 2002; Key, Prager, and Burns, 2015). Farmers can select from a variety of risk management strategies to cope with unforeseen events. The last decade has seen U.S. agricultural policy move toward supporting insurance programs for yield, price, or revenue shortfalls, with the 2014 Farm Act eliminating fixed direct payments (e.g., Effland, Cooper, and O'Donoghue, 2014; O'Donoghue et al., 2016; Shields, 2015). However, some puzzles remain. One such puzzle is that, while most farmers have been found to be risk averse (e.g., Lin, Dean, and Moore, 1974; Dillon and Scandizzo, 1978; Binswanger, 1981; Antle, 1987), they are typically not willing to pay much for crop insurance (e.g., Hazell, Pomareda, and Valdes, 1986; Smith and Glauber, 2012). While most farmers face large risk, why are they are so unwilling to participate in unsubsidized crop insurance? One possible explanation is that they see their own risk management strategies to be good substitutes for insurance. Another puzzle involves the management of agricultural price risk. While futures and option markets for many agricultural commodities are now well established, it remains difficult to explain why so few farmers participate in these markets (e.g., Garcia and Leuthold, 2004, p. 240). While the theory of behavior under risk is

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well-developed,[1] these puzzles indicate the presence of a gap between the theory and the behavioral rules associated with risk management.

This paper is motivated as an attempt to fill this gap. We explore how insurance and other risk management options may interact with one another in agriculture. Addressing this issue requires analyzing the economics of risk management in agriculture. This is challenging to the extent that the empirical investigation of production risk and its welfare effects are complex (e.g., Just and Pope, 2002). The objective of this paper is to take a new look at these issues and to gain new insights on the role of management and risk in agriculture. We define management broadly to include the choice of inputs and outputs affecting farm income and risk exposure. We also evaluate the role played by nonfarm income and its effects on the cost of risk.

This paper studies the measurement of risk exposure and its linkages with management and makes three contributions. Our first contribution is to develop a conceptual approach to the quantification of risk exposure and, based on that measure, an estimation of the cost of risk. The cost of risk is defined as the farmer's willingness to pay to replace a risky payoff by its mean. The analysis is presented under general conditions in which risk preferences depend on the distribution function of payoff. This covers cumulative prospect theory (CPT) (Kahneman and Tversky, 1979; Tversky and Kahneman, 1992), which allows risk preferences to exhibit loss aversion and nonlinear weighting of probabilities. It includes as a special case the expected utility (EU) model. There is growing evidence that CPT provides a better representation of risk preferences than the EU model (see Barberis, 2013, for an overview of the literature), including for farmers (e.g., Liu, 2013; Bocquého, Jacquet, and Reynaud, 2014). Du, Feng, and Hennessy (2017) find that the EU model cannot explain farmers' observed insurance choice. Focusing on income risk, Babcock (2015) finds that neither EU nor CPT can explain farmers' crop insurance behavior.[2] By quantifying risk exposure, conditional on selected management decisions, our paper focuses on the potential role of management. Considering management strategies and insurance as substitutes can help explain why farmers do not express a greater willingness to insure.

Our conceptual contribution is to develop a microeconomic model that includes both price and production risk and general risk preferences using the directional distance function proposed by Luenberger (1995) and Chambers, Chung, and FÃd're (1996) to represent the production technology. This representation applies to multi-output production processes typically found in agriculture. Temporal changes in the directional distance function measure changes in the production frontier. Measures of such changes are used to evaluate technological change and production risk (i.e., the productivity effects of unanticipated shocks). Going beyond previous literature, we investigate the role of both inputs and outputs in risk exposure and risk management.

Our second contribution involves the empirical assessment of price risk and production risk relying on quantile regression. Quantile regression provides a flexible representation of conditional distribution functions (Koenker, 2005), allowing us to examine how management and technology affect the distribution of production risk. Using directional distance measures and applied to a multi-output production process, the analysis documents how inputs and outputs affect productivity under production risk. Previous research has found that such effects can be significant (e.g., Just and Pope, 2002). For example, Just and Pope (1979) found that fertilizer use tends to increase production risk

[1] The economic literature on Pareto-efficient allocations under risk/uncertainty is extensive. Using a state-contingent approach, Debreu (1959, p. 100) showed that, under Pareto efficiency and perfect markets (including risk markets), production decisions under risk/uncertainty are consistent with profit maximization. In this case, efficient firm-level production decisions under risk/uncertainty are independent of consumer/risk preferences (Chambers, 2000; Chambers and Voica, 2017). But frictions in financial markets mean that risk markets are typically incomplete, in which case risk and risk aversion affect production decisions (e.g., Sandmo, 1971; Lin, Dean, and Moore, 1974; Pope and Chavas, 1994). The existence of "puzzles" in agricultural risk management reflects both imperfections in risk markets and complexities in risk management (Just and Pope, 2002).

[2] Babcock (2015) also found that loss aversion can help explain farmers' insurance behavior under loss aversion if loss is perceived when insurance indemnity is less than the premium paid. Note that this is a rather peculiar definition of loss (e.g., compared to loss defined in terms of income risk).

(i.e., that fertilizer is "risk increasing"). Our paper is apparently the first to use quantile regression to evaluate risk in a multi-input, multi-output context. Our analysis provides a framework to evaluate how inputs and outputs affect production risk. This is an important step in the assessment of the role of management in agricultural risk.

Our third contribution is an application to a sample of farms. Our framework is applied to U.S. farm-level data from the U.S. Department of Agriculture Agricultural Resource Management Survey (ARMS) over the period 1996–2011. The annual survey provides detailed information on farm inputs and outputs of selected farms.[3] Risk preferences can vary considerably across individuals (e.g., Halek and Eisenhauer, 2001; Dohmen et al., 2011; Liu, 2013; Bocquého, Jacquet, and Reynaud, 2014; Barseghyan et al., 2018). Without panel data, unobserved heterogeneity makes it difficult to estimate individual risk preferences using secondary data.[4] Such issues constrain our empirical analysis and lead us to restrict our evaluation to selected specifications of risk preferences.

The analysis considers six netputs: two outputs and four inputs. The two outputs are: (i) crops and (ii) livestock. Both outputs are measured as implicit quantities defined as revenue divided by the corresponding price index. The four inputs are: (i) variable inputs, (ii) land, (iii) labor and (iv) capital (excluding land). Variable inputs and capital are measured as implicit quantities. We find that variable inputs contribute to increasing risk, while livestock contributes to reducing risk. We also show how nonfarm income reduces the cost of risk. These effects can be large, indicating that farmers have options in managing their risk exposure. When management has large effects on risk exposure, farm management strategies can behave as a substitute for insurance, providing a possible explanation for why farmers do not express more interest in participating in crop insurance schemes.

Decisions under Risk

Consider a decision maker choosing a m-netput vector $\boldsymbol{z} = (z_1, \ldots, z_m)$ under risk. We use the netput notation, in which outputs in z are positive while inputs are negative. The risk is represented by a random vector $\boldsymbol{e} \in \Omega$, where Ω is the set of states of nature. In agriculture, there are two important sources of risk: production risk, due to unanticipated weather shocks and damages from pests and weeds, and price risk, due to volatile agricultural markets. In our analysis, the state e represents both price risk and production risk facing the decision maker.

The technology associated with netputs z under state e is given by the feasible set $T(e,t) \subset \mathbb{R}^m$, where t is a technology index. Under state e and technology index t, denote the state-contingent netput decision by $\boldsymbol{z}(e,t):\ \Omega\mathbb{R} \rightarrow \mathbb{R}^m$. In this context, $\boldsymbol{z}(e,t) \in T(e,t)$ means that the state-contingent decision $\boldsymbol{z}(e,t)$ is feasible under state $e \in \Omega$ and technology t. Any change in the technology index t captures technological change. For netputs that are chosen *ex ante* (e.g., land allocation decisions made before weather and market conditions are known), the feasible set $T(e,t)$ would restrict the corresponding netputs to be the same for all states e in Ω. Alternatively, for netputs that vary with state e (e.g., crop yield that depends on weather conditions), the feasible set $T(e,t)$ represents how the production possibility set would change with state of nature e.[5] In this context, having $T(e,t) \subset T(e',t)$ means that e' is a more favorable state than e (as e' expands the possibility set relative to e). Throughout, we assume that the set $T(e,t)$ is closed and has an upper bound for all $e \in \Omega$ and all t.

[3] Note that these data are a repeated cross section and not a longitudinal (panel) dataset (as different farms are sampled every year).

[4] Also, without panel data, we do not observe each farmer over time. This prevents us from evaluating either stochastic dynamics or dynamic risk preferences (e.g., as done by Epstein and Zin, 1991; Chavas and Thomas, 1999). In addition, there is evidence that risk and time preferences are not very stable over time (e.g., Chuang and Schechter, 2015). Thus, while exploring risk management in a dynamic context is of interest (e.g., Farrin, Miranda, and O'Donoghue, 2016), this issue is beyond the scope of this paper.

[5] While the set $T(e,t)$ provides a general representation of the technology for a given(e,t), it does not allow possibilities of substitution across states.

Let $\boldsymbol{g} = (g_1, \ldots, g_m) \in \mathbb{R}^m_+$ be a nonstochastic reference bundle of netputs satisfying $\boldsymbol{g} \neq 0$. Following Luenberger (1995) and Chambers, Chung, and FÃd're (1996), for a given g, e, and t, the productivity of the decision $\boldsymbol{z}(e,t)$ can be evaluated using the directional distance function,

$$(1) \qquad D(\boldsymbol{z}(e,t),e,t) = \begin{Bmatrix} \max_\beta \{\beta : (\boldsymbol{z}(e,t) + \beta \boldsymbol{g}) \in T(e,t)\} \text{ if a maximum exists} \\ -\infty \text{ otherwise} \end{Bmatrix}.$$

For a given $\boldsymbol{z}(e,t)$, state e, and technology t, the directional distance function $D(\boldsymbol{z}(e,t),e,t)$ provides a measure of the distance between $\boldsymbol{z}(e,t)$ and the upper bound of the feasible set $T(e,t)$, distance measured in number of units of the reference bundle $\boldsymbol{g}$.

The properties of the directional distance function $D(\boldsymbol{z}(e,t),e,t)$ in equation (1) have been analyzed by Luenberger (1995) and Chambers, Chung, and FÃd're (1996). First, $\boldsymbol{z}(e,t) \in T(e,t)$ implies that $D(\boldsymbol{z}(e,t),e,t) \geq 0$ (since $\beta = 0$ is feasible in equation 1). In this context, having $D(\boldsymbol{z}(e,t),e,t) = 0$ means that point $\boldsymbol{z}(e,t)$ is technically efficient as it located on the upper bound of the feasible set $T(e,t)$. Alternatively, having $D(\boldsymbol{z}(e,t),e,t) > 0)$ implies that point $\boldsymbol{z}(e,t)$ is technically inefficient (as it is located below the production possibility frontier). Second, under free disposal, the directional distance function $D(\boldsymbol{z}(e,t),e,t)$ provides a complete characterization of the technology in the sense that $\{\boldsymbol{z} : D(\boldsymbol{z}(e,t),e,t) \geq 0\} = T(e,t)$. Third, these properties hold for a multi-input, multi-output technology.

For a given e and t, let $p(e,t) = (\boldsymbol{p}_1(e,t), \ldots, p_m(e,t)) \in \mathbb{R}^m_{++}$ be the vector of netput prices under state $e \in \Omega$. This reflects that, besides representing production risk, the states of nature also include unanticipated price shocks. Using the netput notation (where outputs are positive and inputs are negative), for a given state-contingent decision $\boldsymbol{z}(e,t)$, net income under state e and technology t is $[\boldsymbol{p}(e,t) \times \boldsymbol{z}(e,t)] = \sum_{i=1}^m p_i(e,t) z_i(e,t)$. Here, $[p(e,t) \times \boldsymbol{z}(e,t)]$ measures net revenue (i.e., revenue minus cost), the elements of $\boldsymbol{z}$ being positive for outputs (corresponding to revenue) and negative for inputs (corresponding to cost).

Treating the state of nature e as a random vector, let $F(\eta|\boldsymbol{z}(\cdot,t),t) = \Pr_{e\in\Omega}[p(e,t)\boldsymbol{z}(e,t) \leq \eta]$ be the (possibly subjective) cumulative probability distribution of net income conditional on $\boldsymbol{z}(\cdot,t)$ and t. We consider the case in which the decision maker has a preference function, represented by the utility functional $V(F(\cdot))$. We assume that functional $V(F)$ is continuous and nonsatiated in income. Nonsatiation means that $V(F^b) > V(F^a)$ for any $F^a \neq F^b$ satisfying $F^a(\eta) \geq F^b(\eta)$ for all $\eta \in \mathbb{R}$, corresponding to first-order stochastic dominance under any rightward shift in the distribution function from F^a to F^b.

The preference functional $V(F(\cdot))$ has many models as special cases. This includes the rank-dependent utility model, in which $V(F(\cdot)) = \int_\eta U(\eta) dG(F(\eta|\boldsymbol{z}(\cdot,t),t)$, where $U(\eta)$ is a von Neumann–Morgenstern utility function reflecting risk preferences and $G(F) : [0,1] \to [0,1]$ is a strictly increasing function satisfying $G(0) = 0$ and $G(1) = 1$. When $G(F) \neq F$, the model corresponds to cumulative prospect theory (Tversky and Kahneman, 1992), which allows for nonlinearity in probabilities (e.g., Kahneman and Tversky, 1979; Quiggin, 1982, 1993; Hong, 1983; Gonzalez and Wu, 1999; Abdellaoui, 2000; Barberis, 2013). The model also includes as a special case the expected utility model when $V(F(\cdot)) = \int_\eta U(\eta) dF(\eta|\boldsymbol{z}(\cdot,t),t)$ (i.e., when preferences are linear in probabilities and the objective function reduces to $EU(\eta)$, where E is the expectation operator).

For a given t, the optimal choice of state-contingent netputs $\boldsymbol{z}(e,t)$ is given by

$$(2) \qquad \max_{\boldsymbol{z}(\cdot t)} \{V(F(\cdot)) : F(\eta) = \Pr_{e\in\Omega}[\boldsymbol{p}(e,t) \times \boldsymbol{z}(e,t) \leq \eta], \eta \in \mathbb{R}; \boldsymbol{z}(e,t) \in T(t), e \in \Omega,$$

which has solution $\boldsymbol{z}^*(\cdot,t)$. In what follows, we explore the implications of risk and technology for the optimal netput choice $\boldsymbol{z}^*(\cdot,t)$ given in equation (2).

The Cost of Risk

To assess the cost of risk, we need to relate netput choice with its efficiency implications. The previous discussion suggests considering evaluating net income at the technically efficient point $[\boldsymbol{z}(e,t) + D(\boldsymbol{z}(e,t),e,t)\boldsymbol{g}]$. This seems reasonable to the extent that, under nonsatiation, the producer would never want to choose netputs that are technically inefficient. Under technical efficiency, the associated payoff is the adjusted net income $\pi = \eta + D(\boldsymbol{z}(e,t),t)(\boldsymbol{p}(e,t)\boldsymbol{g})$, where $\eta = \boldsymbol{p}(e,t) \times \boldsymbol{z}(e,t)$ and $\boldsymbol{p}(e,t) \times \boldsymbol{g} = \sum_{i=1}^{m} p_i(e,t)g_i$. We start with an alternative characterization of optimal behavior.

We obtain the following key result (see the proof in the Appendix):

PROPOSITION 1. *The optimization problem in equation* (2) *satisfies*

$$\max_{\boldsymbol{z}(\cdot,t)}\{V(F(\cdot)) : F(\eta) = \Pr_{e\in\Omega}[\boldsymbol{p}(e,t) \times \boldsymbol{z}(e,t) \leq \eta], \eta \in \mathbb{R}, \boldsymbol{z}(e,t) \in T(t), e \in \Omega\}$$

$$= \max_{\boldsymbol{z}(\cdot,t)}\{V(F'(\cdot)) : F'(\pi) = \Pr_{e\in\Omega}[\boldsymbol{p}(e,t) \times (\boldsymbol{z}(e,t) + D(\boldsymbol{z}(e,t),e,t)g) \leq \pi], \pi \in \mathbb{R}\}, \tag{3a}$$

where $F'(\pi)$ is the distribution function of adjusted net income:

$$\pi = \boldsymbol{p}(e,t) \times \boldsymbol{z}(e,t) + D(\boldsymbol{z}(e,t),e,t)(\boldsymbol{p}(e,t) \times \boldsymbol{g}). \tag{3b}$$

Proposition 1 shows an alternative way to write the optimal choice $\boldsymbol{z}^*(\cdot,t)$ that solves equation (2). It shows that $\boldsymbol{z}^*(\cdot,t)$ can be equivalently obtained as the solution to the maximization problem stated on right side of equation (3a). Importantly, the feasibility constraint $\boldsymbol{z}(e,t) \in T(e,t)$ does not appear in this equivalent formulation, having been replaced by adding the term $[D(\boldsymbol{z}(e,t),e,t)(\boldsymbol{p}(e,t) \times \boldsymbol{g})]$ to net income (as shown in equation 3b). Proposition 1 states that optimal behavior can be represented by the right side of equation (3a) (i.e., by the maximization of the utility functional over adjusted net income π). Importantly, under general risk and risk preferences, Proposition 1 indicates that knowing the distribution function of $D(\boldsymbol{z}(e,t),e,t)$ provides all the relevant information needed to assess the exposure to production risk.[6] We rely on this result in our subsequent analysis.

Next, we examine welfare measurements under risk. Let $F'(\pi|\boldsymbol{z}(\cdot,t),t) = \Pr_{e\in\Omega}[\boldsymbol{p}(e,t) \times (\boldsymbol{z}(e,t) + D(\boldsymbol{z}(e,t),e,t)\boldsymbol{g})) \leq \pi]$ be the probability distribution of adjusted net income π conditional on $\boldsymbol{z}(\cdot,t)$ and t. And let $F^0(k)$ be a distribution function where all the probability mass is located on point k.

DEFINITION 1. *For a given t and $\boldsymbol{z}(\cdot,t)$, the certainty equivalent associated with a distribution function $F'(\cdot|\boldsymbol{z}(\cdot,t))$ is the sure income CE satisfying*

$$V(F'(\pi|\boldsymbol{z}(\cdot,t),t) = V(F^0(CE)). \tag{4}$$

Denote the solution of equation (4) by $CE(\boldsymbol{z}(\cdot,t),t)$. The certainty equivalent $CE(\boldsymbol{z}(\cdot,t),t)$ is a sure monetary amount that provides a convenient welfare measure under risk. Indeed, under nonsatiation, using equation (4) and substituting equation (3a) into equation (2), the maximization problem in equation (2) is equivalent to maximizing the certainty equivalent $CE(\boldsymbol{z}(\cdot,t),t)$. For a given technology index t, it follows that the optimal netput choice $\boldsymbol{z}^*(e,t)$ in equation (2) can be alternatively written as the solution to the optimization problem

$$CE^*(t) = \max_{\boldsymbol{z}(\cdot,t)}\{CE(\boldsymbol{z}(\cdot,t),t)\}, \tag{5}$$

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[6] To illustrate, consider the special case where $\boldsymbol{g} = (1,0,\ldots,0)$ and z_1 is the first output. Letting $\boldsymbol{z} = (z_1, \boldsymbol{z}_a)$, where $\boldsymbol{z}_a = (z_2,\ldots,z_m)$, it follows that $D(\boldsymbol{z}(e,t),e,t) = f(\boldsymbol{z}_a(e,t),e) - z_1(e,t)$, where $f(\boldsymbol{z}_a(e,t),e,t) = \max_{z_1(e,t)}\{z_1(e,t) : (z_1(e,t),\boldsymbol{z}_a(e,t)) \in T(e,t)\}$ is the classical production frontier for z_1 given $\boldsymbol{z}_a$, e and t. Letting $\boldsymbol{p} = (p_1, \boldsymbol{p}_a)$, where $\boldsymbol{p}_a$ are the prices of z_a, net adjusted income in equation (3b) becomes $\pi = p_1(e,t)f(\boldsymbol{z}_a(e,t),e,t) + \boldsymbol{p}_a(e,t) \times \boldsymbol{z}_a(e,t)$, which is revenue from z_1 (when z_1 is located on the production function $f(\boldsymbol{z}_a(e,t),e,t)$) plus net income associated with $\boldsymbol{z}_a$.

where $CE^*(t)$ is a monetary welfare measure of the decision maker.

DEFINITION 2. *For a given t and* $\mathbf{z}(\cdot,t)$)*, the implicit cost of risk is the risk premium* $R(\mathbf{z}(\cdot,t),t)$ *satisfying*

$$R(\mathbf{z}(\cdot,t),t) = M(\mathbf{z}(\cdot,t),t) - CE(\mathbf{z}(\cdot,t),t), \tag{6}$$

where $CE(\mathbf{z}(\cdot,t),t)$ *is defined in equation* (4)*,* π *is adjusted net income as defined in equation* (3b)*, and*

$$M(\mathbf{z}(\cdot,t),t) \equiv \int_{e\in\Omega} \pi dF(\pi|\mathbf{z}(\cdot,t),t) \tag{7}$$

is the mean of income π*.*

Under the state-contingent decision rule $\mathbf{z}(\cdot,t)$ and technology t, the risk premium $R(\mathbf{z}(\cdot,t),t)$ in equation (6) is the sure amount of money the decision maker is willing to pay to replace random income π by its mean, M. The sign of $R(\mathbf{z}(\cdot,t),t)$ can be used to characterize the risk preferences of the decision maker. In general, under decision rule $\mathbf{z}(\cdot,t)$, the decision maker is said to be $\left\{\begin{array}{c}\text{risk averse}\\ \text{risk neutral}\\ \text{risk lover}\end{array}\right\}$ when $R(\mathbf{z}(\cdot,t),t)\left\{\begin{array}{c}>\\ =\\ <\end{array}\right\}0$. The expected utility model is obtained as a special case. Indeed, under the expected utility model, equations 3, (4), and (6) become $EU(\pi) = U(E(\pi) - -R)$, where E is the expectation operator based on the distribution function $F(\cdot|\mathbf{z}(\cdot,t))$. In this context, R in equation (6) reduces to the Arrow–Pratt definition of the risk premium, where risk aversion corresponds to a concave utility function $U(\pi)$ (e.g., Pratt, 1964).

Combining equations (5) and (6), it follows that the optimal netput choice $\mathbf{z}^*(e,t)$ in equation (2) or (5) can be alternatively written as the solution to the optimization problem

$$CE^*(t) = \max_{\mathbf{z}(\cdot,t)} \{M(\mathbf{z}(\cdot,t),t) - R(\mathbf{z}(\cdot,t),t)\}. \tag{8}$$

Under technology index t, equation (8) identifies efficient decisions that maximize the certainty equivalent of the decision maker under risk. Evaluated at the optimal $\mathbf{z}^*(e,t)$, it includes two terms: expected income $M(\mathbf{z}(\cdot,t),t)$ minus the cost of risk, as measured by $R(\mathbf{z}(\cdot,t),t)$. In general, the cost for risk $R(\mathbf{z}(\cdot,t),t)$ depends on risk exposure, the netput choice $\mathbf{z}(\cdot,t)$, and technology t.

Empirical Assessment of Risk

The usefulness of the previous analysis relies on knowing the distribution function of payoff and the utility functional representing risk preferences. We now discuss empirical methods that can provide a flexible and practical way to estimate the distribution function of payoff, including both production risk and price risk. While the previous discussion allows for objective as well as subjective probability assessments, our empirical analysis is limited to "objective probabilities," assessed from recorded sample information.[7] The specification of risk preferences is discussed subsequently.

[7] Our focus on objective probabilities reflects in large part data constraints from our farm sample (discussed subsequently). Indeed, we do not have a good empirical basis to evaluate subjective probabilities for three reasons: (i) as we are using nonpanel data over the period 1996–2011, we do not know how the farmers in our sample may have perceived information about the economic environment during this period; (ii) there is evidence that many farmers update their information in a way inconsistent with Bayesian learning (e.g., Barham et al., 2015); and (iii) given the evidence that different farmers learn at different rates (Barham et al., 2015, as presented by), modeling learning heterogeneity among farmers is a significant challenge that goes beyond the scope of this paper. On that basis, our empirical risk assessment (including both price risk and production risk) relies on probabilities assessed from recorded sample information.

Production Risk

Our analysis of production risk relies on the directional distance function $D(\boldsymbol{z}(e,t),e,t)$ defined in equation (1). As stated in Proposition 1, the directional distance function provides all the relevant information about technology to assess optimal decisions under risk. On that basis, we proceed estimating the directional distance function $D(\boldsymbol{z}(e,t),e,t)$. The analysis relies on observations of production decisions made by different farms at different periods. Let $\boldsymbol{z}_{jt} \in \mathbb{R}^m$ denote the m-vector of netputs chosen by the jth farm at time t.[8] We consider the case where a set of n_t farmers is observed at time t, all facing similar agroclimatic conditions (i.e., similar soil types and similar weather conditions). In this context, letting $N_t = \{1,\ldots,n_t\}$, we assume that all netput decisions in $\{\boldsymbol{z}_{jt} : j \in N_t\}$ are made at time t under the same technology and the same state of nature e_t. To support this assumption, the empirical analysis that follows focuses on a set of farms in a single crop reporting district (CRD). Letting t represent both time and a technology index, it follows that the observations in $\{\boldsymbol{z}_{jt} : j \in N_t\}$ can be used to estimate $D(\boldsymbol{z}(e_t,t),e_t,t)$ as a representation of the underlying technology $T(e_t,t)$.

The empirical analysis relies on a nonparametric approach, where the feasible set is represented by the smallest set that includes all observations in $\{\boldsymbol{z}_{jt} : j \in N_t\}$ (e.g., Varian, 1984; Coelli et al., 2005).[9] The nonparametric approach has several advantages: (i) It provides a flexible representation of the technology; (ii) it does not require making any parametric assumptions; (iii) it is not subject to endogeneity bias (as it involves no parameter to estimate); and (iv) it is easy to implement empirically.

We consider the following nonparametric representation of the technology $T(e_t,t)$:

$$T^e(e_t,t) = \max_{\lambda}\{\boldsymbol{z} : \boldsymbol{z} \leq \sum_{k\in N_t} \lambda_k \boldsymbol{z}_{kt};\ \sum_{k\in N_t} \lambda_k = 1; \lambda_k \geq 0, k \in N_t, \tag{9}$$

where $T^e(e_t,t)$ is the smallest convex set that includes all observations in $\{\boldsymbol{z}_{jt} : j \in N_t\}$ under variable returns to scale (Varian, 1984; Coelli et al., 2005). Investigations based on equation (9) have been called data envelopment analysis (DEA) or frontier analysis. As t changes, $T^e(e_t,t)$ can change due to the shocks e_t (reflecting risk effects that vary over time) as well as the technology index t (reflecting technological change).

Substituting equation (9) into equation (1), evaluated at point $z \in \mathbb{R}^m$, we obtain

$$D^e(z,e_t,t) = \max_{\beta,\lambda}\{\beta : \boldsymbol{z} + \beta \boldsymbol{g} \leq \sum_{k\in N_t} \lambda_k \boldsymbol{z}_{kt};\ \sum_{k\in N_t} \lambda_k = 1; \lambda_k \geq 0, k \in N_t\}, \tag{10}$$

where $D^e(\boldsymbol{z},e_t,t)$ measures the number of units of $\boldsymbol{g}$ separating point $\boldsymbol{z}$ from the upper bound of the set $T^e(e_t,t)$. Obtaining $D^e(\boldsymbol{z},e_t,t)$ is relatively easy: Finding the maximum in equation (10) is a simple linear programming problem.

Next, consider that farms located in a given agroclimatic region are observed over time. Assume that we have observations on $\{\boldsymbol{z}_{jt} : j \in N_t\}$ over S periods. Let $\tau \equiv \{t_1,t_2,\ldots,t_S\}$ be the set of periods. In general, $D^e(\boldsymbol{z},e_t,t)$ in equation (10) reflects the effects of management z, shocks e_t, and technology t. Treating the shocks e as random variables with a given probability distribution, it follows that, conditional on (z,t), $D^e(\boldsymbol{z},e_t,t)$ has the conditional cumulative probability distribution function $H_D(c|\boldsymbol{z},\ t) = \Pr[D^e(\boldsymbol{z},e_t,t) \leq c]$.

This conditional probability distribution can be evaluated as follows: First, choose a set of netputs $Z = \{\boldsymbol{z}^1,\ldots,\boldsymbol{z}^n\}$ reflecting n alternative management schemes. Second, using equation (10), evaluate $D^e(\boldsymbol{z},e_t,t)$ for all $\boldsymbol{z} \in Z$ and all $t \in \tau$. Third, define the conditional quantile $Q_D(q|\boldsymbol{z},t)$ as the inverse of the conditional distribution $H_D(c|\boldsymbol{z},t) : Q_D(q|\boldsymbol{z},t) = \inf_c\{c : H_D(c|\boldsymbol{z},t) \geq q\}$, where q

[8] Note that this does not require observing the same farmers over time. In other words, our analysis holds with or without panel data.

[9] An alternative approach would be to use a parametric approach, where the function $D(z(e_t,t),e_t,t)$ is assumed to take a parametric form (e.g., Aigner, Lovell, and Schmidt, 1977; Kumbhakar and Lovell, 2000; Coelli et al., 2005).

denotes the qth quantile satisfying $q \in (0,1)$. Assume that $Q_D(q|\boldsymbol{z},t) = \boldsymbol{X}(\boldsymbol{z},t)\boldsymbol{\alpha}(q)$, where $\boldsymbol{X}(\boldsymbol{z},t)$ is a vector of explanatory variables (that depend on $\boldsymbol{z}$ and t) and $\boldsymbol{\alpha}(q)$ is a vector of parameters associated with the qth quantile, $q \in (0,1)$. Then, the parameters $\boldsymbol{\alpha}(q)$ can be estimated by quantile regression (Koenker, 2005). By allowing the parameters $\boldsymbol{\alpha}(q)$ to vary across quantiles q, quantile regression provides a flexible representation of (conditional) distribution functions. Under some regularity conditions, the quantile estimates $\boldsymbol{\alpha}^e(q)$ are consistent and have an asymptotic normal distribution (Koenker, 2005). These estimates can then be used to obtain the quantile function $Q_D^e(q|\boldsymbol{z},t) = \boldsymbol{X}(z,t)\alpha^e(q)$. Fourth, inverting the quantile function $\boldsymbol{Q}_D^e(q|\boldsymbol{z},t)$ generates $H_D^e(c|\boldsymbol{z},t)$, a consistent estimate of the distribution function $H_D(c|\boldsymbol{z},t)$.

Proposition 1 establishes that knowing the distribution function of $D(\boldsymbol{z}(e,t),e,t)$ provides all the relevant information to assess the exposure to production risk. Since the previous estimation method gives an estimate of the probability distribution of $D(\boldsymbol{z},e,t)$, it provides an empirical basis to investigate exposure to production risk and its linkages with management.

Price Risk

When the production process takes place over an extended period of time, input decisions are made before outputs and output prices are known. This can expose producers to significant output price risk. In this case, there is a need to assess the exposure to price risk. Empirically, this can be done by using historical data in the estimation of the price distribution. At time t, consider the case where the ith output price p_i evolves according to the stochastic difference equation $p_{i,t} = h_i(p_{i,t-1},u_t,t)$, where u_t is an independently distributed random variable. Then, the probability distribution of the ith output price at time t is $H_i(c|p_{i,t-1},t) = \Pr[h_i(p_{i,t-1},u_t,t) \leq c]$. Consider its inverse, the associated conditional quantile $Q_i(q|p_{i,t-1},t) = \inf_c\{c : H_i(c|p_{i,t-1},t) \geq q\}$, where $q \in (0,1)$. Following Koenker and Xiao (2006), assume that prices are driven by a quantile autoregressive process, where $Q_i(q|p_{i,t-1},t) = \gamma_{i,0}(q) + \gamma_{i,1}(q)p_{i,t-1} + \gamma_{i,2}(q)t$. Then, using historical data on p_{it}, quantile estimation can generate consistent estimates of the parameters $\gamma_i(q)$ (Koenker, 2005; Koenker and Xiao, 2006). These estimates can be used to obtain consistent estimates of the quantile function, $Q_i^e(q|p_{i,t-1},t)$, and its inverse, $H_i^e(q|p_{i,t-1},t)$. Again, by allowing the parameters $\gamma_i(q)$ to vary across quantiles q, a quantile estimator provides a flexible representation of the marginal distribution function of the ith price, $H_i^e(q|p_{i,t-1},t)$.

Assessing the Joint Distribution of Risk

Typically, producers face both price risk and production risk. As discussed previously, production risk can be assessed using the conditional distribution function $H_D^e(c|z,t)$. And price risk can be evaluated using the marginal distribution function of each output price $H_i^e(q|p_{i,t-1},t)$. Next, we need a joint assessment of risk across all sources of risk. This requires establishing linkages between marginal distributions and the joint distribution of risk. This can be done using a copula. From Sklar's (1959) theorem, any k-dimensional joint distribution function $H(x_1,\ldots,x_k) = \Pr[X_1 \leq x_1,\ldots,X_k \leq x_k]$ can be expressed in terms of its marginal distributions $H_i(x_i) = \Pr[X_i \leq x_i], i = 1,\ldots,k$, as

$$H(x_1,\ldots,x_k) = C(H_1(x_1),\ldots,H_k(x_k)), \tag{11}$$

where $C : [0,1]^k \to [0,1]$ is a copula. Thus, a copula provides a general way to link a joint distribution to their associated marginal distributions (Nelsen, 1999). As we illustrate next, this can support an empirical analysis of risk exposure in the presence of multiple sources of risk.

Table 1. Summary Statistics ($N = 1,484$)

Netputs	Mean	Median	Standard Deviation
Crop output	18.84	10.50	42.09
Livestock output	159.71	67.46	256.79
Variable inputs	127.06	62.24	184.49
Land (acres)	206.53	140.00	235.86
Labor (hours)	15,374.00	3,640.00	34,905.00
Capital	1,641.61	1,162.22	1,637.13
Age (years)	54.62	52.51	13.43

Notes: With the exception of land and labor, netputs are measured as implicit quantities, calculated as the ratio of revenue (or expense) to the appropriate price index.

An Application to Agriculture

Our main focus is to investigate the effects of farm management and technology on risk exposure. In this context, the decision maker is a farmer exhibiting risk preferences given by the utility function $V(F)$, where F is the distribution function of farm payoff reflecting both price risk and production risk.

Data

Our analysis relies on data from the Agricultural Resource Management Survey (ARMS) covering U.S. farms over the period 1996–2011. The survey is sponsored jointly by the USDA's Economic Research Service (ERS) and National Agricultural Statistics Service (NASS). The annual survey provides detailed information on farm inputs and outputs as well as farm business and household characteristics. The data used for this analysis are drawn from the Cost and Returns Report version of ARMS, which is the "phase III" farm-level survey.[10] Note that the survey draws a different sample of farm operators every year, so the data form a repeated cross section and not a panel.

Our analysis studies a subset of the data, focusing on farms located in the South-Central CRD in Wisconsin, which is a part of the U.S. Corn Belt. Farms in this CRD all face similar agroclimatic conditions. Our analysis considers six netputs ($m = 6$): two outputs and four inputs. The two outputs are: (i) crops and (ii) livestock. The four inputs are: (iii) variable inputs, (iv) land, (v) labor, and (vi) capital (excluding land). Land is measured in acres of harvested land. Labor is the number of hours worked on the farm. The other netputs are measured as implicit quantities (i.e., as values [$ thousands] divided by corresponding price indices). The price indices for capital, variable inputs, crops, and livestock are obtained from the ERS (http://www.ers.usda.gov/data-products/agricultural-productivity-in-the-us).

The data are first evaluated for outliers. Any observation that involves a variable more than 10 times the value of its 0.8 quantile is treated as an outlier and is deleted. As a result, 153 observations were dropped, resulting in a total of 1484 observations on farm inputs and outputs over the period 1996–2011. Table 1 reports summary statistics for the data.

Assessment of Production Risk

An important step in our analysis is the assessment of production risk. As discussed previously, this involves estimating the distribution function of the directional distance function $D(z, e, t)$ conditional on (z, t). This is done following the nonparametric approach discussed previously. We choose the reference bundle to be $\boldsymbol{g} = (g_1, g_2, 0, 0, 0, 0)$, where (g_1, g_2) is set equal to the sample mean of outputs (implying that we set $(g_3, \ldots, g_6) = 0$). This means that the directional distance function

[10] Additional details on the structure and design of ARMS and copies of the questionnaires are available at https://www.ers.usda.gov/data-products/arms-farm-financial-and-crop-production-practices/documentation.

Table 2. Quantile Estimation of the Conditional Distribution of Production Risk Represented by $D^e(\cdot)$ in Equation (10), Selected Quantiles q

Variable	$q = 0.1$	$q = 0.3$	$q = 0.5$	$q = 0.7$	$q = 0.9$
Intercept	−23.82***	−29.68**	−36.11**	−50.61**	−59.61*
Crop output	11.69***	15.56***	13.65***	10.86***	1.31
Livestock output	0.30	−0.22	−1.07***	−1.70**	−3.17***
Variable input	−0.72*	−1.09**	0.19	1.34	4.98**
Land	73.13	550.30***	666.80***	1,000.99***	1,807.89***
Labor	1.10	13.45***	24.24***	29.99***	36.67***
Capital	0.04	0.07**	0.06	0.10	0.08
t	0.01***	0.015***	0.018***	0.025***	0.03***

Notes: Single, double, and triple asterisks (*, **, ***) indicate significance at the 10%, 5%, and 1% level.

$D(\mathbf{z}, e, t)$ can be interpreted as measuring proportional changes in mean outputs. Alternatively, the outputs being goods sold in competitive markets, our directional distance function $D(\mathbf{z}, e, t)$ measures proportional changes in mean gross revenue. In our analysis, the time/technology index t covers 16 periods $(S = 16)$, from 1996 to 2011. Holding z constant, changes in the directional distance function over time capture changes in the production frontier, providing a basis to evaluate the effects of unanticipated production shocks e and of technological change (as reflected in changes in t). To assess the effects of management, we choose $\mathbf{z} \in Z$, where $Z = \{\mathbf{z}^1, \ldots, \mathbf{z}^{n_{1996}}\}$ and $\mathbf{z}^j = [\mathbf{z}_{j,1996} + D^e(z_{j,1996}, e_{1996}, 1996)\mathbf{g}], j \in N_{1996}$, N_{1996} being the set of sample farms observed in 1996. In this context, $\mathbf{z}^j$ is a technically efficient point associated with the jth farm observed in 1996. Thus, choosing $\mathbf{z} \in Z$ corresponds to choosing technically efficient management schemes in 1996. This guarantees that our evaluation points remain within the range of our data. Conditional on technology t, we study the impact of management on production risk by examining how netputs $\mathbf{z} \in Z$ affect the distribution of temporal changes in the directional distance function.

Following the method discussed previously, we use quantile regression to estimate the distribution function of $D^e(\mathbf{z}, e, t)$ obtained from equation (10), conditional on (z, t). Table 2 reports the quantile regression results for selected quantiles $q = (0.1, 0.3, 0.5, 0.7, 0.9)$. The results indicate that both management (as reflected by input and output choices) and technology (as reflected by the time-trend index t) have significant effects on the distribution function.

We conduct a series of hypothesis tests on the quantile regression model (see Table 3). First, we test whether the regression parameters $\alpha(q)$ vary across quantiles q. Testing the null hypothesis that $\boldsymbol{\alpha}(0.1) = \boldsymbol{\alpha}(0.3) = \boldsymbol{\alpha}(0.5) = \boldsymbol{\alpha}(0.7) = \boldsymbol{\alpha}(0.9)$, the F-value was 10.74, with a p-value of 0.001. Thus, we find strong statistical evidence that the regression parameters vary across quantiles. Second, we test whether technological change (as represented by t) plays a role. Testing the null hypothesis that the coefficient of t is 0, the p-values were consistently < 0.001 for quantiles $q = (0.1, 0.3, , 0.5, 0.7, 0.9)$. Thus, technological change is found to have significant effects on the distribution of production risk. Third, we test whether management affects the distribution of production risk. Testing the null hypothesis that the coefficients of the management variables (inputs and outputs) are all 0, the p-values were all <0.001 for quantiles $q = (0.1, 0.3, , 0.5, 0.7, 0.9)$. Thus, we find strong statistical evidence that management plays a major role, as it affects exposure to production risk. This key result and its implications will be discussed further.

The estimates from quantile regression presented in Table 2 provide useful information. First, the time trend variable t has a coefficient of 0.018 at the median ($q = 0.5$), which corresponds to a rate of technological change of about 1.8% per year, consistent with previous estimates for U.S. agriculture (e.g., Ball et al., 1997). Interestingly, Table 2 shows that the coefficient of t is always positive, indicating that technological progress contributes to a rightward shift in the distribution of production risk. But the shift is smaller in the lower tail of the distribution and higher in the upper tail, implying that technological progress shifts the upper tail of the distribution faster than it does

Table 3. Hypothesis Testing about the Distribution of Production Risk

Hypothesis Test	*F* Value	Degrees of Freedom	*p*-Value
H_0: same parameters across quantiles (0.1, 0.3, 0.5, 0.7, 0.9)	10.74	(28, 2612)	$< 0.001^{***}$
H_0: no technical change			
$q = 0.1$	100.97	(1, 520)	$< 0.001^{***}$
$q = 0.3$	93.72	(1, 520)	$< 0.001^{***}$
$q = 0.5$	111.39	(1, 520)	$< 0.001^{***}$
$q = 0.7$	43.34	(1, 520)	$< 0.001^{***}$
$q = 0.9$	57.43	(1, 520)	$< 0.001^{***}$
H_0: management has no effect			
$q = 0.1$	7.62	(6, 520)	$< 0.001^{***}$
$q = 0.3$	19.44	(6, 520)	$< 0.001^{***}$
$q = 0.5$	16.77	(6, 520)	$< 0.001^{***}$
$q = 0.7$	25.62	(6, 520)	$< 0.001^{***}$
$q = 0.9$	37.94	(6, 520)	$< 0.001^{***}$

Notes: Triple asterisks (***) indicate significance at the 1% level.

the lower tail. Therefore, holding management z constant, median productivity rises and the spread of the distribution of production risk increases (i.e., exposure to production risk rises).

Second, Table 2 shows the effects of inputs on production risk. While increasing variable inputs implies a rise in median productivity, it also amplifies risk exposure (as we will discuss further). The increased spread can be seen in Table 2 through the effects of variable inputs on the lower tail and upper tail of the distribution. This is consistent with the analysis of Just and Pope (1979) applied to fertilizer. From Table 2, increases in land, labor, or capital are found to shift the whole probability distribution to the right. But for land and labor, the shifts are faster in the upper tail, indicating that—while these inputs contribute to improving median productivity—they also increase the skewness of production risk (by increasing the spread of the distribution in the upper tail).

Table 2 also shows the effects of the output mix (crops vs. livestock) on production risk. Crop output is found to contribute to shifting the distribution function to the right. In contrast, livestock tends to shift the distribution to the left, especially in its upper tail. The distribution shifts reported in Table 2 reflect basic characteristics of agriculture in the Corn Belt. Crops are more productive than livestock, but they are also riskier (as crops increase exposure to "downside risk" located in the lower tail of the distribution). And livestock is less risky: Livestock activities reduce exposure to "upside risk" (located in the upper tail of the distribution) while not increasing exposure to downside risk.

Our analysis is also used to evaluate the distribution of farm income: After re-estimating the quantile regression model for all quantiles $q \in (0,1)$, the parameter estimates can be utilized to simulate the distribution of $D^e(\mathbf{z}, e_t, t)$: $H^e(c|\mathbf{z}, t) = \Pr[D^e(\mathbf{z}, e_t, t) \leq c]$ (as discussed previously).

Evaluated at 2005 prices, $[(p_{2005}g)D^e(\mathbf{z}, e_t, t)]$ gives a measure of gross revenue obtained under netputs z and technology t. Using our quantile estimates of $D^e(\mathbf{z}, e_t, t)$, distribution functions of this gross revenue are reported in Figure 1 under alternative scenarios. Figures 1a and 1b include simulations of the effects of technological change and alternative management scenarios. Figure 1a considers three situations: $t = 2005$ (treated as the base case), $t = 2000$, and $t = 2010$. Comparing the distribution functions for different t documents the extent and nature of technological change. Figures 1a and 1b also show how alternative management schemes affect farm income under production risk by simulating the effects of an 80% increase in each netput use (denoted by "netput+" in Figure 1). The effects of an 80% increase in crops and livestock are presented in Figure 1a. And the effects of an 80% increase in variable inputs, land, labor, and capital are presented in Figure 1b.

As expected, Figure 1a confirms that technological progress generates a strong rightward shift in

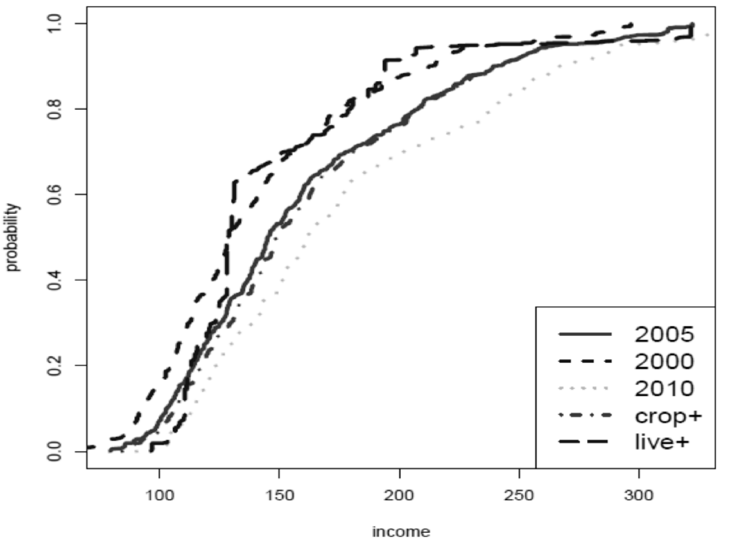

Figure 1a. Estimated Probability Distribution of Farm Income $[(p_{2005}\ g)\ D(z, e_t, t)]$ **under Alternative Scenarios for Selected Years, Crop and Livestock**

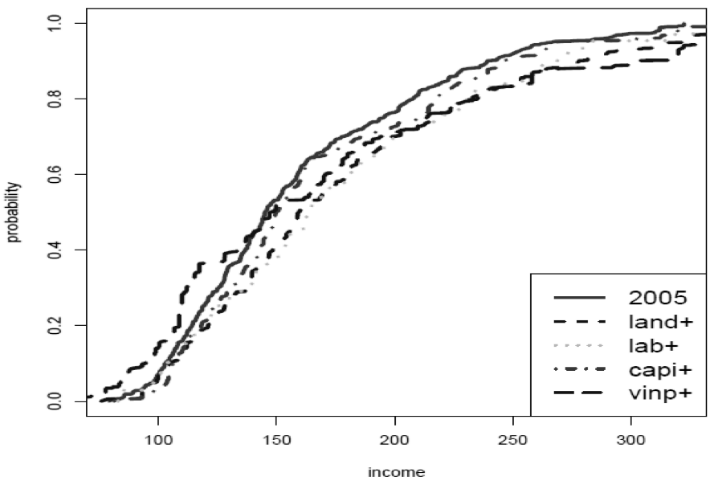

Figure 1b. Estimated Probability Distribution of Farm Income $[(p_{2005}\ g)\ D(z, e_t, t)]$ **under Alternative Scenarios for 2005 and Selected Inputs**

the distribution of farm income. It also shows that livestock is less productive but also less risky than crops. And Figure 1b shows that variable inputs are risk-increasing (as they increase exposure to both "upside risk" and "downside risk"). Figure 1 illustrates the large effects of technological progress: The change in technology between 2000 and 2010 generates a larger shift in median farm income than an 80% increase in any netput.

Assessment of Price Risk

As discussed previously, the empirical assessment of price risk relies on a quantile autoregression model applied to output prices. Historical prices for crops and livestock were obtained from the USDA. The parameter estimates are reported in Table 4 for selected quantiles, $q = (0.1, 0.3, 0.5, 0.7, 0.9)$. Table 4 shows that crop prices exhibit more dynamics than livestock

Table 4. Estimation of the Distribution of Price Risk, AR(1) and Quantile Regression

Panel A: Crop Price, p_{1t}

		Quantile Regression, Selected Quantiles				
Variables	**AR(1)**	$q = 0.1$	$q = 0.3$	$q = 0.5$	$q = 0.7$	$q = 0.9$
Intercept	0.122	0.266	0.303	0.142	−0.282	−0.278
$p_{1,t-1}$	0.768***	0.614**	0.600*	0.783*	1.145***	1.287***
t	0.020**	0.010	0.011	0.012	0.027*	0.017*
	$R^2 = 0.788$					

Panel B: Livestock Price, p_{2t}

		Quantile Regression, Selected Quantiles				
Variables	**AR(1)**	$q = 0.1$	$q = 0.3$	$q = 0.5$	$q = 0.7$	$q = 0.9$
Intercept	0.634***	0.758	0.607*	0.519	0.892**	0.431
$p_{2,t-1}$	0.090	−0.073	0.160	0.238	−0.173	0.465
t	0.023***	0.017	0.014	0.023*	0.028***	0.019**
	$R^2 = 0.732$					

Notes: Single, double, and triple asterisks (*, **, ***) indicate significance at the 10%, 5%, and 1% level.

prices. The quantile estimates of the effects of the time trend t indicate that the upper tails of the price distributions shift up over time, indicating a rise in price risk for both crops and livestock. Going beyond Table 4, quantile estimates of output prices were obtained for all quantiles $q \in (0,1)$. The results were used to estimate the marginal distribution functions of output prices, providing an empirical assessment of output price risk in the analysis that follows.

Assessment of Income Risk

Using the estimates $D^e(z, e_t, t)$ from equation (10), consider farm income

$$v(z, e_t, t) = \sum_{k=1}^{3} p_k(e_t)[z_k + D^e(z, e_t, t) g_k], \tag{12}$$

where $p_1(e_t)$ is the price of crops, $p_2(e_t)$ is the price of livestock, and p_3 is the price of variable inputs in 2005. Thus, $v(\boldsymbol{z}, e_t, t)$ in equation (12) measures farm income net of variable input costs under both price and production risk.

As discussed previously, we use a copula to link the marginal distribution functions to the joint distribution function, including production risk, and price risk for both crops and livestock. We assume that the copula $C(H_1, H_2, \ldots)$ in equation (11) has a Gaussian distribution (Nelsen, 1999; Arbenz, 2013). Under a Gaussian distribution, the matrix of correlation coefficients ρ gives sufficient statistics for the copula. In this context, the density function of a Gaussian copula is $\frac{1}{\sqrt{\det(\rho)}} \exp\{-0.5[u_1, u_2, \ldots][\rho^{-1} - I][u_1, u_2, \ldots]^T\}$, where $u_i = \Phi^{-1}(H_i), i = 1, 2, \ldots$, Φ^{-1} being the inverse standard normal distribution function (Nelsen, 1999; Arbenz, 2013). The correlation coefficients in ρ reflect the stochastic linkages between the price of crops, the price of livestock, and production risk. Empirical evidence indicates that such correlations vary among commodities, across space, and over time (e.g., Harwood et al., 1999; Tejeda and Goodwin, 2009). Previous research has found that production risk and price risk tend to be negatively correlated (Harwood et al., 1999; Dismukes and Coble, 2006) and that crop price and livestock price tend to be positively correlated (Tejeda and Goodwin, 2009). On that basis, our analysis assumes the correlation coefficients to be −0.3 between production risk and crop price, −0.1 between production risk and livestock price; and +0.2 between crop price and livestock price. These correlation coefficients were then used to parametrize the copula $C(H_1, H_2, \ldots)$ in equation (11), generating an estimated representation of income risk across all states, including both production risk and price risk for crops and livestock. In turn, this was used to simulate the distribution function of income $v(z, e_t, t)$ in equation (12) under alternative scenarios involving management $z \in Z$ and technologya $t \in T$. Such simulations provide the basis for the risk analysis that follows.

Risk Preferences

Analyzing the cost of risk depends on risk preferences. Following the discussion presented previously, risk preferences are represented by the function $V(F(\cdot)) = \int_{e \in \Omega} U(\cdot) dG(F(\cdot|\mathbf{z}(\cdot, t), t)$. Let total income be $(w + v) > 0$, where w is nonfarm income and v is farm income given in equation (12). We consider the following risk preferences:

$$U(w+v) = \left\{ \begin{matrix} \frac{(w+v)^{1-r_1}-1}{1-r_1} - \frac{(w+L)^{1-r_1}-1}{1-r_1} \\ k[\frac{(w+v)^{1-r_2}-1}{1-r_2} - \frac{(w+L)^{1-r_2}-1}{1-r_2}] \end{matrix} \right\} \text{ if } \left\{ \begin{matrix} v \geq L \\ v < L \end{matrix} \right\}, \tag{13a}$$

where L is a given threshold point and $k \in \mathbb{R}_{++}]$. Following Prelec (1998),

$$G(F) = \exp[-(-\log(F))^{\alpha}], \tag{13b}$$

where $\alpha \in (0, 1]$ is a probability weight. When $k = 1$ and $\alpha = 1$, equations (13a) and (13b) reduce to the expected utility (EU) model. In addition, when $r_1 = r_2 = r$, equations (13a) and (13b) become an EU model under constant relative aversion (CRRA), with r as the Arrow–Pratt relative risk aversion coefficient and $EU(w + v) = \int_{e \in \Omega} \frac{(w+v)^{1-r}-1}{1-r} dF(e|z(\cdot, t), t)$ (Pratt, 1964; Arrow, 1965). As argued by Gollier (2001), CRRA has been commonly assumed in the analysis of risk aversion, the coefficient of relative risk aversion r being typically in the range from 0 (corresponding to risk neutrality) to 5 (corresponding to a high level of risk aversion). On that basis, we report our welfare evaluation for $r = 1.5$, corresponding to a moderate level of risk aversion.[11] When $k \neq 1$ and $\alpha \in (0, 1)$, equations (13a) and (13b) correspond to a non-EU preference function in two ways. First, including $k > 1$ in equation (13a) introduces a kink in the utility function at the threshold point L, allowing for "loss aversion" as risk preferences can differ between favorable outcomes (when $v \geq L$) and unfavorable outcomes (when $v < L$) (Kahneman and Tversky, 1979; Liu, 2013). Second, including $\alpha < 1$ in equation (13b) introduces nonlinearity in the probabilities: The $G(F)$ function then has an inverted S-shape overweighting the probability of rare events (see Tversky and Kahneman, 1992; Prelec, 1998; Neilson, 2003; Fehr-Duda and Epper, 2012; Barberis, 2013). Liu (2013) and Bocquého, Jacquet, and Reynaud (2014) present evidence that farmers tend to overweight such probabilities.

In what follows, we report results under two risk preference scenarios. The first scenario corresponds to expected utility (EU) preferences, where $r_1 = r_2 = r = 1.5$, $k = 1$, and $\alpha = 1$. The second scenario involves cumulative prospect theory (CPT), where $r = 1.5$, $k = 2$, $L = 50$, and $\alpha = 0.8$. The choice of $\alpha = 0.8$ is consistent with the empirical evidence presented in Liu (2013) and Bocquého, Jacquet, and Reynaud (2014). The threshold point $L = 50$ is located within the payoff range but below the mean payoff in 2005. This CPT scenario illustrates how departures from the EU model can affect the cost of risk.[12]

[11] The simulation analysis was also done under different levels of risk aversion r. As expected, a higher (lower) risk aversion coefficient r increased (reduced) the cost of risk.

[12] We also conducted a sensitivity analysis on the CPT parameters (α, k, L). The results showed how the simulation results can be affected by these parameters. The results (not reported here) are available from the authors upon request.

Table 5. Cost of Risk R and R/M under Alternative Management Scenarios

	Expected Utility, *EU* $r = 1.5,\ \alpha = 1,\ k = 1$		Cumulative Prospect Theory, *CPT* $r = 1.5,\ \alpha = 0.8,\ k = 2$	
Scenarios	R_{EU}	R_{EU}/M	R_{CPT}	R_{CPT}/M
Year 2005	11.40	0.115	14.92	0.151
Year 2010	13.10	0.111	15.72	0.134
Crop+	10.52	0.101	12.87	0.124
Livestock+	3.61	0.021	4.87	0.029
Variable input+	26.88	0.549	37.99	0.776
Land+	14.74	0.127	18.04	0.156
Labor+	14.98	0.129	19.00	0.164
Capital+	11.88	0.113	14.40	0.136
Ninc+	9.42	0.079	11.51	0.097

Estimates of the Cost of Risk

We use the quantile estimates reported previously to evaluate the distribution function of $D(\mathbf{z}, e, t)$ conditional on $(\mathbf{z}, t)$. These estimates are used to estimate farm income v in equation (12) and mean household income $M(\mathbf{z}, t) = E(w + v)$. After replacing π by $(w + v)$, the certainty equivalent $CE(\mathbf{z}, t)$ is obtained from equation (4) and the cost of risk $R(\mathbf{z}, t)$ from equation (6). The results are reported in Table 5 under selected scenarios evaluating the role of technology t and management z.[13] Table 5 also reports the relative cost of risk R/M, measuring the proportion of mean income a farmer would be willing to pay to eliminate all production and price risk.

First, Table 5 reports that, evaluated at means values under 2005 conditions, R/M amounts to 11.5% of mean income under EU and 15.1% of mean income under CPT. This shows that the cost of risk can be relatively large, reflecting the importance of agricultural risk. In turn, this indicates a strong willingness to insure against income risk either under EU or CPT. This result is consistent with the analysis presented by Babcock (2015). Table 5 reports that prospect theory increases the farmer's willingness-to-pay to eliminate risk relative to a more traditional EU specification, although the differences between R_{EU} and R_{CPT} is not very large. Table 5 also reports that, depending on management, the relative cost of risk R/M varies from 0.021 to 0.776. Thus R amounts to between 2% and 78% of expected net income, indicating that the cost of risk can vary considerably across management scenarios. The relative cost of risk R/M is lowest (0.021, or 2.1%) under the scenario (livestock+, EU), reflecting that livestock is a low-risk enterprise. And R/M is highest (0.776, or 77.6%) under the scenario (Variable Input+, CPT), reflecting that variable inputs are risk increasing.[14] This documents our key result: Management can have sizable effects on the cost of risk. As such, management strategies can provide effective options to reduce risk exposure and the associated cost of risk. Our analysis indicates that neglecting these options would provide an incomplete evaluation of the economics of risk in agriculture.

Second, comparing $t = 2005$ with $t = 2010$, Table 5 shows that technology has a positive effect on the cost of risk R. Between 2005 and 2010, R_{EU} rises from 11.40 to 13.10 (or 14.9%) and R_{CPT} rises from 14.92 to 15.72 (or 5.3%). Yet between 2005 and 2010, mean income rises at least as fast as the cost of risk, implying that the relative cost of risk does not increase: R/M stays at about 0.11% under EU, and it declines from 0.151 to 0.134 under CPT. This is another important result:

[13] Unless otherwise indicated, the evaluations are conducted at sample means, with $w = 30$ and $t = 2005$.

[14] This does not imply that farmers should produce more livestock or use lower levels of variable inputs. Indeed, the cost of risk R captures only a part of the economic motivation for a particular management strategy.

While technological change has increased both mean income and income risk, the effects of new agricultural technologies have not increased the relative cost of risk R/M.

Third, Table 5 reports the effects of management z on the cost of risk R. Recall that "netput+" denotes situations involving an 80% increase in the corresponding netput. Table 5 reports that alternative management schemes have major impacts on the cost of risk R. At one extreme, livestock offers the least amount of production risk: Under "livestock+," the relative cost of risk R/M declines to 0.115 to 0.021 under EU and from 0.151 to 0.029 under CPT. Again, this documents that livestock production involves much lower risk exposure than crops. At the other extreme, variable inputs can contribute to increasing production risk: under "variable netput+," the relative cost of risk R/M rises sharply from to 0.115 to 0.549 under EU and from 0.151 to 0.776 under CPT. Again, it means that variable inputs are "risk increasing."

Finally, Table 5 reports another scenario, denoted by "Ninc+," where nonfarm income w is increased by 20 ($20,000). Table 5 shows that increasing nonfarm income reduces the cost of risk R from 11.40 to 9.42 (or -17.3%) under EU and from 14.92 to 11.51 (or -29.6%) under CPT. This reflects important income effects: Under CRRA, the farmer exhibits decreasing absolute risk aversion and the cost of risk declines with wealth (Pratt, 1964). This is another key result: Our analysis shows that nonfarm income can contribute to significant reductions in the cost of risk (and thus in the incentive to insure).

The cost of risk R reported in Table 5 measures the willingness-to-pay to eliminate risk. But risk elimination is typically not feasible. Next, we evaluate the welfare effects associated with marginal reductions of exposure to downside risk as typically done in insurance schemes. Letting Q_i be the ith quantile of income π, we consider $\pi_i = \max\{\pi, Q_i\}$ as the income received after eliminating the exposure to income risk below Q_i and moving the associated probability mass to Q_i. This is the scenario of a stylized subsidized insurance scheme, in which the farmer does not pay for the insurance contract while benefiting from the associated risk reduction. Denote the mean income of π_i by $M(Q_i)$, which is an increasing function of Q_i. We also consider the case where income is $[\pi_i + M(Q_0) - M(Q_i)]$, corresponding to a fair insurance contract that eliminates the risk below Q_i while keeping expected income constant. This is the scenario in which the farmer pays for insurance, keeping expected income constant, the entire benefit of the insurance scheme coming from the associated risk reduction. Holding management constant, we apply our analysis to the evaluation of risk under these alternative insurance schemes. Evaluating the effects of eliminating risk below the ith quantile, we denote the cost of risk by $R(Q)$ under subsidized insurance and by $R'(Q)$ under fair insurance. The results are presented in Table 6 for both EU and CPT risk preferences under selected quantiles.

As expected, Table 6 shows that eliminating the risk below quantile Q_i always increases expected income $M(Q_i)$. These effects can be substantial: $M(Q_i)$ goes from 98.99 at Q_0 to 110.14 (+11.4%) at $Q_{0.5}$. By reducing exposure to risk, increasing Q_i also reduces the cost of risk. Table 6 indicates that these effects are notable.

First, consider the case of subsidized insurance, where $\pi_i = \max\{\pi, Q_i\}$. Table 6 shows that R_{EU} declines from 11.40 at Q_0 to 8.08 at $Q_{0.3}$ and to 5.10 at $Q_{0.5}$. And R_{CPT} declines from 14.92 at Q_0 to 8.24 at $Q_{0.3}$ and to 4.41 at $Q_{0.5}$. Finding that R_{CPT} differs from R_{EU} indicates that the willingness to insure can vary under CPT compared to EU. Table 6 reports the relative cost of risk defined as the ratio R/M (i.e., the cost of risk R as a proportion of expected income M). Figure 2 reports the relationships between the relative cost of risk R/M and the level of risk protection that varies with Q_i. Based on subsidized insurance schemes $\pi_i = \max\{\pi, Q_i\}$, R_{EU}/M declines from 0.115 at Q_0 to 0.079 at $Q_{0.3}$ and to 0.046 at $Q_{0.5}$. This demonstrates that the cost for risk decreases when insurance provides greater risk protection. And R_{CPT}/M declines from 0.151 at Q_0 to 0.040 at $Q_{0.5}$. As showed in Figure 2, the relative cost of risk R/M follows similar patterns with respect to q under expected utility EU versus CPT.

Alternatively, under fair insurance schemes, where income is $[\pi_i + M(Q_0) - M(Q_i)]$, Table 6 shows that the relative cost of risk R'_{EU}/M declines from 0.115 at Q_0 to 0.056 at $Q_{0.5}$. And R'_{CPT}/M

Table 6. Cost of Risk R under Alternative Income Insurance Scenarios: $R(Q_i)$ for Subsidized Insurance and $R'(Q_i)$ for Fair Insurance

Quantile $Pr[\pi \leq Q_i] = i$	Q_0	$Q_{0.2}$	$Q_{0.3}$	$Q_{0.4}$	$Q_{0.5}$	$Q_{0.6}$	$Q_{0.7}$
Mean payoff $M(Q_i) = E[max(\pi, Q_i)]$ (\$thousands)	98.99	100.32	102.70	105.99	110.14	116.94	129.05
Cost of risk under EU, $r = 1.5, \alpha = 1, k = 1$							
$R_{EU}(Q_i)$	11.40	9.82	8.08	6.50	5.10	3.52	1.81
$R'_{EU}(Q_i)$	11.40	9.94	8.36	6.89	8.38	6.75	4.72
$R_{EU}(Q_i)/M(Q_i)$	0.12	0.10	0.08	0.06	0.05	0.03	0.01
$R'_{EU}(Q_i)/M(Q_i)$	0.12	0.10	0.08	0.07	0.06	0.04	0.02
Cost of risk under CPT, $r = 1.5, \alpha = 0.8, k = 2$							
$R_{CPT}(Q_i)$	14.92	10.86	8.24	6.10	4.41	2.68	1.02
$R'_{CPT}(Q_i)$	14.92	10.99	8.53	6.52	4.91	3.23	1.50
$R_{CPT}(Q_i)/M(Q_i)$	0.15	0.11	0.08	0.08	0.04	0.02	0.01
$R'_{CPT}(Q_i)/M(Q_i)$	0.15	0.11	0.09	0.07	0.05	0.03	0.02

Notes: $M(Q_i) = E[max(\pi, Q_i)]$ is the mean payoff obtained after eliminating the risk below the ith quantile Q_i, the ith quantile of π. The cost of risk R is obtained under two insurance scenarios: $R(Q_i)$ under subsidized insurance, where $\pi_i = max\{\pi, Q_i\}$; and $R'(Q_i)$ under fair insurance, where income is $[\pi_i + M(Q_0) - M(Q_i)]$. The estimates are evaluated at $t = 2005$ and at sample means.

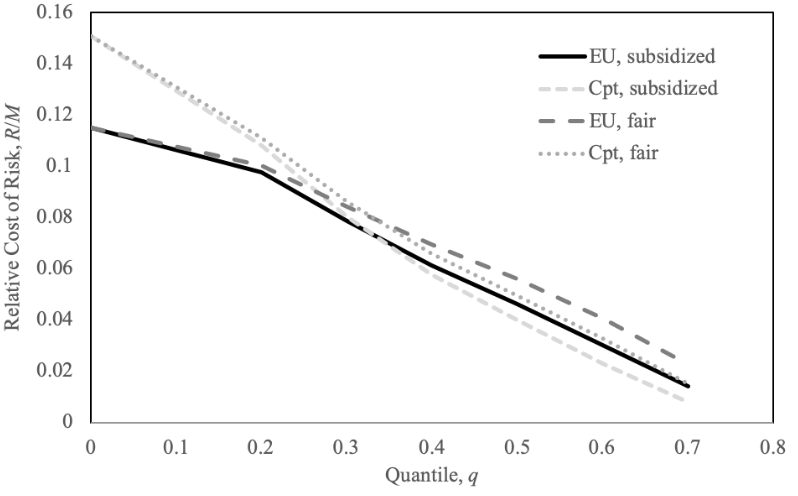

Figure 2. Relative Cost of Risk under Alternative Insurance Schemes $R(Q_i)/M(Q_i)$ under Two Scenarios: Subsidized Insurance and Fair Insurance

declines from 0.151 at Q_0 to 0.050 at $Q_{0.5}$. The differences between subsidized insurance and fair insurance are due to income effects: Keeping expected income constant reduces income under the fair insurance scheme, implying a relative increase in the cost of risk (under CRRA). Table 6 reports that these income effects are present but of moderate magnitude.

In addition, from Table 6, note that any risk reduction in the range below $Q_{0.3}$ gives a relatively large decline in the cost of risk R compared to a corresponding increase in mean payoff M. This would correspond to insurance coverage for catastrophic events. Thus, within the range below $Q_{0.3}$,

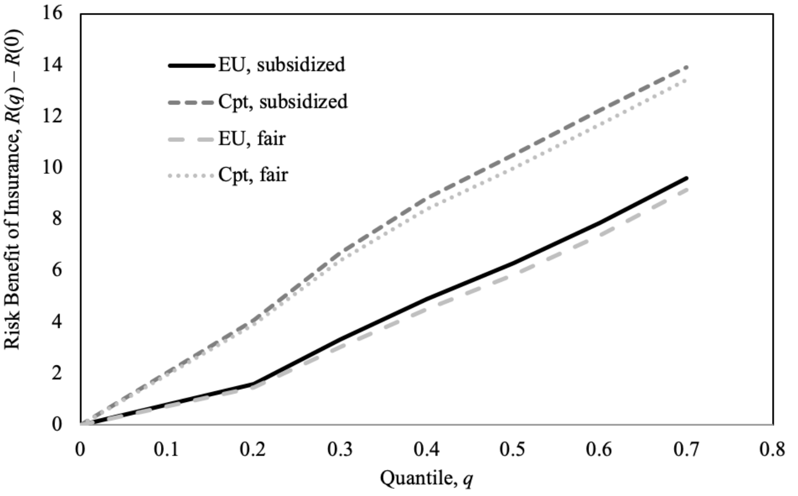

Figure 3. Risk Benefit of Alternative Insurance Schemes $[R(Q_0) - R(Q_i)]$ under Two Scenarios: Subsidized Insurance and Fair Insurance

risk redistribution has the potential to generate positive welfare benefits. But any risk reduction in the range above $Q_{0.4}$ generates a larger increase in mean payoff M than a reduction in the cost of risk R. In other words, in situations where $Q_i \geq Q_{0.4}$, the welfare effects of increasing q are dominated by changes in mean payoff M, indicating that the welfare benefit of insurance schemes on the cost of risk can be relatively large, but only when they reduce risk in the lower tail of the payoff distribution.

Finally, note that the risk benefit of an insurance scheme eliminating risk below q can be written as $[R(Q_0) - R(Q_i)]$ under subsidized insurance and as $[R'(Q_0) - R'(Q_i)]$ under fair insurance. Estimates of these risk benefits are reported in Figure 3, which shows that the risk benefits from insurance are higher under CPT compared to EU. In other words, EU would underestimate the risk benefit of insurance compared to CPT for both subsidized insurance and fair insurance. Our sensitivity analysis on the parameters (a, k, L) indicates that this result is due in part to the overweighing of the probability of rare events. When the risk eliminated by insurance corresponds to low-probability events in the lower tail of the distribution, CPT would put more weight on the probability of these events, leading a higher willingness to insure under CPT than EU. These results are similar to the ones obtained by Babcock (2015). They indicate that, in the evaluation of income risk under risk aversion, neither EU nor CPT can explain why farmers do not express more willingness to buy insurance.

Comparing the results presented in Figure 3 and Table 5 is instructive. As noted previously, Table 5 shows that reducing variable inputs or increasing livestock has large and negative impacts on the cost of risk R. Importantly, such impacts are similar in magnitude to the risk benefits of insurance reported in Figure 3. This is one of our key results: When management has large effects on risk exposure, farm management strategies can behave as substitute for insurance, thus providing a possible explanation for why farmers do not exhibit a stronger willingness to participate in crop insurance programs.

Conclusion

This research has developed an analysis of risk exposure and the cost of risk. We developed a conceptual model under general conditions. The assessment of production risk relies on the directional distance function. Quantile regression is used to investigate the role of management and its effects on risk exposure. Finally, the conceptual analysis of the cost of risk is presented under general risk preferences. This analysis is applied to a sample of U.S. farms in the Corn

Belt. It documents how technology and management can affect risk exposure. We find that variable inputs are risk-increasing but that livestock is risk-reducing and that nonfarm income contributes to reducing the cost of risk. These effects can be large. Our analysis shows that farmers have many options in managing their risk exposure. Neglecting such management options (e.g., in the evaluation of insurance) would provide an incomplete view of risk behavior. Our analysis helps refine our understanding of the role of management in risk exposure. By showing how farm management strategies can reduce risk exposure, we document that management and insurance can behave as substitutes, thus providing a possible explanation for why farmers do not express greater willingness to participate in crop insurance.

Our analysis focused on agriculture in a part of the U.S. Corn Belt, but it could be extended in several directions. First, our empirical results are specific to a given agroclimatic region. There is a need to expand the analysis to other agroclimatic regions. Second, risk preferences can vary across individuals (e.g., Halek and Eisenhauer, 2001; Dohmen et al., 2011; Liu, 2013; Bocquého, Jacquet, and Reynaud, 2014; Barseghyan et al., 2018). We have not addressed the implications of heterogeneity in individual risk preferences. When imperfectly observed, such heterogeneity makes it difficult to evaluate risk preferences and their welfare implications. More research is needed on this topic. Third, the analysis presented in this paper was limited to a static approach. It would be useful to explore risk management in a dynamic framework. Finally, current concerns about climate change make the analysis presented in this paper highly relevant. Will technological progress and improved management be sufficient to deal with the increased climate-induced-risk in agriculture? More research is needed to examine this issue.

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Appendix:

Proof of Proposition 1. From equation (1), since $\boldsymbol{z}(e,t) \in T(e,t)$ implies that $D(\boldsymbol{z}(e,t),e,t) \geq 0$, it follows that $T(e,t) \subset \{\boldsymbol{z}(e,t) : D(\boldsymbol{z}(e,t),e,t) \geq 0\}$. Given $(\boldsymbol{p}(e,t) \times \boldsymbol{g}) > 0$ and under feasibility, we obtain $\pi - \eta = \boldsymbol{p}(e,t) \times (D(\boldsymbol{z}(e,t,t)\boldsymbol{g}) \geq 0$. Thus, under nonsatiation, the distribution function of $F'(\pi)$ weakly dominates the distribution function $F(\eta)$, implying that $V(F'(\cdot)) \geq V(F(\cdot))$. It follows that

$$\max_{\boldsymbol{z}(\cdot,t)}\{V(F(\cdot)) : F(\eta) = \Pr_{e\in\Omega}[\boldsymbol{p}(e,t) \times \boldsymbol{z}(e,t) \leq \eta],\ \eta \in \mathbb{R},\ \boldsymbol{z}(e,t) \in T(t),\ e \in \Omega\}$$

$$\leq \max_{\boldsymbol{z}(\cdot,t)}\{V(F(\cdot)) : F(\eta) = \Pr_{e\in\Omega}[\boldsymbol{p}(e,t) \times \boldsymbol{z}(e,t) \leq \eta],\ \eta \in \mathbb{R}, D(\boldsymbol{z}(e,t),e,t) \geq 0\}$$

$$\leq \max_{\boldsymbol{z}(\cdot,t)}\{V(F'(\cdot)) : F'(\pi) = \Pr_{e\in\Omega}[\boldsymbol{p}(e,t) \times (\boldsymbol{z}(e,t) + D(\boldsymbol{z}(e,t),e,t)\boldsymbol{g}) \leq \pi],\ \pi \in \mathbb{R},\ D(\boldsymbol{z}(e,t),e,t) \geq 0\}\}$$

$$\leq \max_{\boldsymbol{z}(\cdot,t)}\{V(F'(\cdot)) : F^{'(\pi)} = \Pr_{e\in\Omega}[\boldsymbol{p}(e,t) \times (\boldsymbol{z}(e,t) + D(\boldsymbol{z}(e,t),e,t)\boldsymbol{g}) \leq \pi],\ \pi \in \mathbb{R}.\}\}$$

We now need to show that the reverse inequality holds:

$$\max_{\boldsymbol{z}(\cdot,t)}\{V(F(\cdot)) : F(\eta) = \Pr_{e\in\Omega}[\boldsymbol{p}(e,t) \times \boldsymbol{z}(e,t) \leq \eta],\ \eta \in \mathbb{R},\ \boldsymbol{z}(e,t) \in T(t),\ e \in \Omega\}$$

$$\geq \max_{z(\cdot,t)}\{V(F'(\cdot)) : F'(\pi) = \Pr_{e\in\Omega}[\boldsymbol{p}(e,t) \times (\boldsymbol{z}(e,t) + D(\boldsymbol{z}(e,t),e,t)\boldsymbol{g}) \leq \pi], \pi \in \mathbb{R}\}.$$

Let $\boldsymbol{z}^{+}(e,t)$ be the solution to the second optimization. Under nonsatiation and given $\pi - \eta = \boldsymbol{p}(e,t) \times (D(\boldsymbol{z}(e,t),t)\boldsymbol{g})$, this reverse inequality holds when $D(\boldsymbol{z}^{+}(e,t),e,t) = -\infty$ for any state $e \in \Omega$. Next, consider the case where $D(\boldsymbol{z}^{+}(e,t),e,t) > -\infty$ for all $e \in \Omega$. Then, there exists at least one $\boldsymbol{z}'(e,t)$ satisfying $D(\boldsymbol{z}'(e,t),e,t) > -\infty$ for all $e \in \Omega$. From equation (1), we have $[\boldsymbol{z}'(e,t) + D(z'(e,t),e,t)g] \in T(e,t)$, implying that

$$\max_{z(\cdot,t)}\{V(F(\cdot)) : F(\eta) = \Pr_{e\in\Omega}[\boldsymbol{p}(e,t) \times \boldsymbol{z}(e,t) \leq \eta],\ \eta \in \mathbb{R}, \boldsymbol{z}(e,t) \in T(t),\ e \in \Omega\}$$

$$\geq V(F'(\cdot)) \text{ evaluated at } [z'(e,t) + D(z'(e,t),e,t)\boldsymbol{g}],$$

for any $\boldsymbol{z}'(e,t)$ satisfying $D(\boldsymbol{z}'(e,t),e,t) > -\infty$ for all $e \in \Omega$. When $D(\boldsymbol{z}^{+}(e,t),e,t) > -\infty$, choosing $\boldsymbol{z}'(e,t) = \boldsymbol{z}^{+}(e,t)$ gives the desired result. ■