Imperfect Competition between Florida and São Paulo (Brazil) Orange Juice Producers in the U.S. and European Markets

Jeff Luckstead, Stephen Devadoss, and Ron C. Mittelhammer

We develop a strategic trade model to analyze the oligopolistic competition between Florida and São Paulo processors in the U.S. orange juice market and São Paulo processors in the European orange juice market. We obtain analytical results of the effects of changes in trade liberalization. A structural econometric model is derived from the theoretical model, and the new empirical industrial organization literature is used to estimate the market power of Florida and São Paulo producers. We simulate the effects of U.S. and European tariff reductions on prices, quantities, and trade volume.

Key words: market power, NEIO, orange juice, strategic trade policy

Introduction

Orange juice production is highly concentrated both geographically and economically, which can lead to oligopolistic competition in this market. The geographic concentration stems from the locations of the largest orange juice producing regions in the world: Florida in the United States and São Paulo in Brazil.¹ Florida and São Paulo orange juice processors supply on average about 89% of the U.S. market,² while São Paulo processors supply an average of 84% of the European market (U.S. Department of Agriculture, Foreign Agricultural Service, 2012). Florida supplied an average of 92% of all U.S. processed oranges for the period 1986–2010 (Economic Research Service, 2012b). For the same period, an average of 23% of the total U.S. orange juice supply was imported, and São Paulo shipped 74% of all U.S. orange juice imports (Food and Agriculture Organization of the United Nations, 2012).

The economic concentration in the orange juice market arises from few firms operating in Florida and São Paulo. According to the Florida Department of Agriculture and Consumer Services, Division of Fruit and Vegetable Inspection (2012), the number of processors in Florida declined from forty-five in 1997 to sixteen in 2010.³ The total production by the Florida processors in 2010 is 840 million single strength equivalent (SSE) gallons. The U.S. retail price in 2010 is $4.40 per SSE gallon. In São Paulo, three firms produced about 90% of the total supply. São Paulo exports

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¹ Orange juice comprises frozen orange juice and not-from-concentrate orange juice.
² Other countries such as Mexico, Costa Rica, and Belize also supply orange juice. However, the market share of these three countries relative to that of Florida and São Paulo’s is less than 6% (Food and Agriculture Organization of the United Nations, 2012).
³ Orange juice processors are a subset of orange processors. According to Florida Department of Agriculture and Consumer Services, Division of Fruit and Vegetable Inspection (2012), there were thirty-five orange processors in Florida during the 2000–2001 season, and Spreen and Fernandes (2000) report a total of eighteen orange juice processors in Florida during the 2000–2001 season.
171 million SSE gallons to the United States and 518 million SSE gallons to the European Union. The producer price in São Paulo is $1.98 per SSE gallon. The high concentration of processors in Florida and São Paulo makes it possible for these processors to exercise market power by engaging in oligopolistic competition.

Hart (2004) reports that orange juice processors in both countries have high bargaining power with their buyers and exert oligopoly power. But, oligopsony power by juice buyers is unlikely because of the lack of concentration or collusion among final consumers. Orange growers are likely to operate under perfect competition because of the large number of growers. Thus, orange juice processors are the only group in the supply chain with a potential to influence the U.S. or European orange juice price and extract oligopoly rents.

The United States and Europe rank first and second in terms of per capita orange juice consumption in the world (Hart, 2004). Because of the high level of consumption, the United States exports only 6% of its total production (Economic Research Service, 2012b). Thus, São Paulo dominates the European orange juice market. Brazil exports 99% of its processed oranges because Brazilians mainly drink fresh squeezed orange juice (Hart, 2004; Mendes, 2011). Since Europe produces a small amount of orange juice, it accounts for about 80% of total world imports.

The U.S. and European orange juice markets are protected by tariffs. The U.S. citrus juice tariff has insulated juice producers in Florida from overseas competition since 1930. In the United States, the most-favored-nation applied tariff for orange juice was $0.3501 per SSE gallon until 1994 when the Uruguay Round of the General Agreement on Tariffs and Trade mandated that the tariff decrease by 15% to $0.2971 per SSE gallon by 2000 (Brown, Spreen, and Lee, 2004; Spreen, Brewster, and Brown, 2003). Europe imposed an ad valorem tariff of 19% until the Uruguay Round, after which the tariff was reduced to 15.20% by 2000.

With the progress of the Doha Round negotiations, trade liberalization will likely reduce the tariffs in the orange juice market. In addition, if the Free Trade Area of the Americas is successfully negotiated, Florida orange juice processors will face even more competition. LaVigne (2003) states that supporters of free trade argued that removal of the tariff will provide U.S. consumers with the lowest cost orange juice possible. However, LaVigne (2003) also reports that opponents of free trade contend that without the tariff, the already highly concentrated Brazilian processors, will control an even larger U.S. market share, which will leave consumers further exposed to the price setting behavior of oligopoly producers in São Paulo.

Spreen, Brewster, and Brown (2003) developed a spatial equilibrium model of processed oranges and showed that the domestic orange juice price declined by $0.22 per SSE gallon if the U.S. tariff was removed. Brown, Spreen, and Lee (2004) determined that potential unilateral elimination of the U.S. tariff reduced the U.S. orange juice price by $0.22 per SSE gallon, while simultaneous elimination of the U.S., European, and Japanese tariffs lowered the U.S. price by only $0.13 per SSE gallon. Brown (2010) estimated the European demand for orange juice to gain insight into the price response in Europe and found the demand elasticities range from \(-0.45\) to \(-0.69\). Wang, Xiang, and Reardon (2006) analyzed the impact of supply shocks due to adverse weather on oligopolistic competition in the U.S. orange juice market by estimating market power using the grower’s price, quantity, an indicator variable for winter freezes, and a trend in their marginal cost function. Their results show that a supply shock decreases the market power of orange juice processors.

Given the dominance of Florida and São Paulo, we advance the literature by analyzing the market power of orange juice processors of these states using the U.S. retail price and European price. Strategic trade theory analyzes policies implemented by governments to improve their industries’ position in international markets operating under imperfect competition. Brander and Spencer (1985), in their seminal work on strategic trade theory, showed that unlike under perfect competition, an export subsidy can result in a net welfare gain for the home country at the expense of the

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4 Wang, Xiang, and Reardon (2006) reports that there were over 7,500 orange farms in Florida in 2002.
5 This tariff is applied to the imports of orange juice from São Paulo. The price of São Paulo orange juice is $1.98 per SSE gallon.
competitor’s welfare due to rent shifting from the foreign to the home industry. We follow this literature to theoretically analyze the U.S. and European orange juice markets under imperfect competition. We use the New Empirical Industrial Organization (NEIO) literature and draw on several empirical studies that have estimated industry-level market power in an international setting (Yerger, 1996; Lavoie, 2005) for our empirical work.

The specific objectives of this study are to 1) develop a strategic trade model to analyze the imperfect competition of Florida and São Paulo orange juice processors, 2) derive analytical results to theoretically examine the effect of a change in the U.S. and European tariffs on the orange juice market in the United States and Europe, 3) specify and estimate an econometric model based on the theoretical analysis and compute the degree of market power exerted by Florida and São Paulo orange juice processors, and 4) simulate the effect of U.S. and European tariff reductions on prices and quantities in the United States, São Paulo, and Europe.

**Theoretical Analysis**

Based on the above description of the orange juice market, we formulate a strategic trade model under oligopolistic competition and derive the comparative static results of a change in the U.S. and European tariffs.

**Strategic Trade Model**

Consider the U.S. and European orange juice markets: Florida processors sell in the U.S. market and São Paulo processors export to both the United States and Europe. Florida and São Paulo processors face downward sloping demand functions, allowing for the potential to exert market power. São Paulo firms have a distinct cost advantage due to lower input prices but incur transportation costs and face tariffs to export to the United States and Europe. The U.S. and European governments impose tariffs on orange juice imports.

The profit function \((\pi_f)\) for the representative orange juice processor in Florida is

\[
\pi_f = p_u(q_u)q_f - C^f(q_f; x^f) - F^f,
\]

where \(p_u\) is the price of orange juice in the United States, \(p_u(q_u)\) is the U.S. demand for orange juice, \(q_u = q^f + q^{su}\) is total quantity of orange juice sold in the United States, \(q^f\) is the quantity of orange juice sold by Florida processors, \(q^{su}\) is the quantity of orange juice sold in the United States by São Paulo processors, \(C^f(\cdot)\) and \(F^f\) are the variable and fixed costs of production in Florida, and \(x^f\) are the supply shifters. The profit function \((\pi_s)\) for the representative processor in São Paulo is

\[
\pi_s = (p_u(q_u) - \tau^u)q^{su} + \frac{p_e(q^{se})}{(1 + \tau^e)}q^{se} - C^s(q^{su} + q^{se}; x^s) - F^s,
\]

where \(\tau^u\) is the per unit tariff on imports to the United States, \(\tau^e\) is the \textit{ad valorem} tariff on imports to Europe, \(p_e\) is the price of orange juice in Europe, \(p_e(q^{se})\) is the European demand for orange juice produced in São Paulo, \(q^{se}\) is the total quantity of orange juice sold in Europe by São Paulo processors, \(C^s(\cdot)\) and \(F^s\) are the variable and fixed costs of production in São Paulo, and \(x^s\) are the supply shifters.

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6 This is an industry-level analysis, but a firm-level oligopolistic model that is aggregated to describe average firm behavior is at the root of the analysis.
The profit functions are differentiated with respect to $q^f$, $q^{se}$, and $q^{su}$ to derive the first-order conditions that implicitly determine the reaction or best-response functions:

\[ \pi_q^f = \frac{\partial p^f}{\partial q^f} q^f + p^f - \frac{\partial C^f(q^f; x^f)}{\partial q^f} = 0 \]

\[ \pi_q^{su} = \left( \frac{\partial p^{su}}{\partial q^{su}} q^{su} + p^{su} - \tau^{su} \right) - \frac{\partial C^s(q^{su} + q^{se}; x^s)}{\partial q^{su}} = 0 \]

\[ \pi_q^{se} = \frac{1}{1 + \tau^e} \left( \frac{\partial p^{se}}{\partial q^{se}} q^{se} + p^e \right) - \frac{\partial C^e(q^{su} + q^{se}; x^e)}{\partial q^{se}} = 0. \]

The reaction functions imply a unique solution if they are downward sloping and the second-order conditions are met.

**Tariff Analysis**

As elaborated in the introduction, the Uruguay Round agreement reduced the U.S. and European tariffs. Furthermore, once the Doha round is completed, orange juice tariffs will likely be reduced further.

To analyze the effect of changes in U.S. and European tariffs on Florida and São Paulo orange juice production, we totally differentiate the reaction functions (3)–(5) and solve the linear system of equations to obtain the following comparative static results. The impacts of a change in the U.S. tariff, $\tau^u$, are (see the appendix for the derivation).

\[ \frac{dq^{su}}{d\tau^u} = \frac{1}{|A|} \pi_q^f q_u \pi_q^{se} q_u \pi_q^{se} q_u < 0 \]

\[ \frac{dq^{se}}{d\tau^e} = -\frac{1}{|A|} \pi_q^f q_u \pi_q^{se} q_u \pi_q^{se} q_u > 0 \]

\[ \frac{dq^f}{d\tau^u} = \frac{1}{|A|} \pi_q^f q_u \pi_q^{se} q_u \pi_q^{se} q_u > 0. \]

A reduction in the U.S. tariff decreases the price of São Paulo’s orange juice in the U.S. market. As a result, exports from São Paulo to the United States increase (equation 6) at the expense of their exports to Europe (equation 7). The higher imports from São Paulo displace Florida’s orange juice production, we totally differentiate the reaction functions (3)–(5) and solve the linear system of equations to obtain the following comparative static results. The impacts of a change in the U.S. tariff, $\tau^u$, are (see the appendix for the derivation).

\[ \frac{dq^{su}}{d\tau^u} = \frac{1}{|A|} \pi_q^f q_u \pi_q^{se} q_u \pi_q^{se} q_u < 0 \]

\[ \frac{dq^{se}}{d\tau^e} = -\frac{1}{|A|} \pi_q^f q_u \pi_q^{se} q_u \pi_q^{se} q_u > 0 \]

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\[ \frac{dq^{se}}{d\tau^e} = -\frac{1}{|A|} \pi_q^f q_u \pi_q^{se} q_u \pi_q^{se} q_u > 0. \]

\[ \frac{dq^f}{d\tau^u} = \frac{1}{|A|} \pi_q^f q_u \pi_q^{se} q_u \pi_q^{se} q_u > 0. \]

The determinant of $A$ is given by $|A| = \pi_q^f q_u \pi_q^{se} q_u \pi_q^{se} q_u - \pi_q^f q_u \pi_q^{se} q_u \pi_q^{se} q_u - \pi_q^f q_u \pi_q^{se} q_u \pi_q^{se} q_u$ and is positive for plausible supply and demand functions.
A decrease in the European tariff lowers the price of São Paulo’s orange juice in Europe. Consequently, São Paulo exporters divert their exports from the United States to Europe (equations 9 and 10). As exports from São Paulo to the United States decrease, Florida’s orange juice sales in the U.S. market expand (equation 11). This will lower (increase) São Paulo’s (Florida’s) market share in the United States, thereby potentially reducing (increasing) São Paulo’s (Florida’s) market power.

**Empirical Analysis**

In this section, we derive the econometric model based on the strategic trade model, discuss data and sources, present the estimation results, calculate the Lerner index, and present the simulation analysis and results.

**Econometric Model**

Total quantity in the United States is defined as
\[
q_u = q_f + q_{su} + \tilde{q}_o,
\]
where \(\tilde{q}_o\) is the quantity sold by processors other than Florida and São Paulo processors and is exogenous in the model because it is relatively small compared to \(q_f + q_{su}\) (see footnote 2).

We specify the econometric model by rewriting the first-order conditions (3)–(5) from the theoretical model as
\[
\begin{align*}
  p^u &= \frac{\partial C_f(q_f; x_f)}{\partial q_f} + \theta f \varepsilon^u p^u, \\
  p^s &= \frac{\partial C_s(q_{su} + q_{se}; x_s)}{\partial q_{su}} + \tau u + \theta_{su} \varepsilon^u p^u, \\
  p^e &= (1 + \tau e) \frac{\partial C_s(q_{su} + q_{se}; x_s)}{\partial q_{se}} + \theta_{se} \varepsilon^e p^e,
\end{align*}
\]
where \(x_i\)s \((i = f \text{ and } s)\) are supply shifters such as input prices, \(\theta f = \frac{\partial q^u}{\partial q_f} \frac{q_f}{q^u} = 1\) is the conjectural elasticity for Florida processors, \(\varepsilon^u = -\frac{\partial p^u}{\partial q^u} \frac{q^u}{p^u}\) is the U.S. price flexibility of demand, \(\theta_{su} = \frac{\partial q^u}{\partial q_{su}} \frac{q^{su}}{q^u}\) is the conjectural elasticity for São Paulo processors exporting to the United States, \(\varepsilon^e = -\frac{\partial p^e}{\partial q^{se}} \frac{q^{se}}{p^e}\) is the European price flexibility of demand, and \(\theta_{se} = \frac{\partial q^{se}}{\partial q^{se}} \frac{q^{se}}{q^e}\) is the conjectural elasticity for São Paulo processors exporting to Europe, and is not necessarily equal to one because it is based on weighted average of each firm’s conjectural elasticity (see footnote 7, Porter (1983), and Devadoss, Luckstead, and Mittelhammer (2013) for additional analysis).

As seen by the second terms on the right-hand-side of equations (12)-(14), an industry’s ability to set price above marginal cost is driven by the interaction of the conjectural elasticities and demand flexibilities. Four cases are possible. First, under perfect collusion, orange juice processors act as a monopoly, and the conjectural variation and market share are one (\(\partial q^u/\partial q_f = q_f/q^u = 1\)). This implies that the conjectural elasticity is equal to one, and the markup is determined by the demand flexibilities. Second, if the orange juice processors operate under Cournot competition, the conjectural variation is equal to one (\(\partial q^u/\partial q_f = 1\)) and markup depends on the interaction of the demand flexibility and the representative firm’s market share. Third, under a fully flexible market structure, market power is given by the conjectural elasticity weighted by the demand flexibility. Fourth, under perfect competition, the orange juice processors’ market share is small enough so they cannot influence the orange juice price. This implies that the conjectural elasticity and thus markup is zero, and price is equal to the marginal cost.
For estimable supply relations, we first define marginal cost and demand functions. Then, we consider identification of demand and supply parameters and the conjectural elasticities. The marginal cost functions for Florida and São Paulo processors are defined as

\[
\frac{\partial C^f}{\partial q^f} = \beta_0^f + \beta_1^f q^f + \beta^f x^f
\]

\[
\frac{\partial C^s}{\partial q^{su}} = \beta_0^{su} + \beta_1^{su} (q^{su} + q^{se}) + \beta_2 g^u + \beta^{su} x^i
\]

\[
\frac{\partial C^e}{\partial q^{se}} = \beta_0^{se} + \beta_1^{se} (q^{su} + q^{se}) + \beta_2 g^e + \beta^{se} x^i,
\]

where \(\beta_j^i\)'s are marginal cost coefficients, and \(g^u\) and \(g^e\) are, respectively, per unit transport costs of shipping orange juice from Brazil to the United States and Europe. The U.S. and European demand functions are

\[
p^u = \alpha_0^u + \alpha_1^u (q^f + q^{su} + \bar{q}^f) + \alpha^u Z^u
\]

\[
p^e = \alpha_0^e + \alpha_1^e q^{se} + \alpha^e Z^e,
\]

where \(\alpha_j^i\)'s are demand coefficients and \(Z^i\)'s are U.S. and European demand shifters.

Using the first-order conditions (12)–(14), the marginal cost functions (15)–(17), and the demand flexibilities derived from the demand functions (18) and (19), the estimable supply relations for Florida and São Paulo processors are represented as:

\[
p^u = \beta_0^f + \beta_1^f q^f + \beta^f x^f + \theta^f \alpha_1^u (q^f + q^{su} + \bar{q}^f)
\]

\[
p^u = \beta_0^{su} + \beta_1^{su} (q^{su} + q^{se}) + \beta_2 g^u + \beta^{su} x^s + \tau^{su} + \theta^{su} \alpha_1^u (q^f + q^{su} + \bar{q}^f)
\]

\[
p^e = (1 + \tau^e) (\beta_0^{se} + \beta_1^{se} (q^{su} + q^{se}) + \beta_2 g^e + \beta^{se} x^s) + \theta^{se} \alpha_1^e q^{se}.
\]

The parameters in the U.S. demand function (18) are identified if the number of excluded exogenous variables from the demand function is greater than or equal to the number of endogenous variables \(p^u, q^f, \) and \(q^{su}\). The parameters in the U.S. supply relation (20), including \(\theta^f \alpha_1^u\), are identified if the number of excluded exogenous variables is greater than or equal to the number of endogenous variables \(p^u, q^f, \) and \(q^{su}\). Estimation of the system of equations will yield an estimate for \(\alpha_1^u\) and a combined estimate for \(\theta^f \alpha_1^u\). Using the estimate of \(\alpha_1^u\), the conjectural elasticity \(\theta^f\) can be identified from the estimate of \(\theta^f \alpha_1^u\). Devadoss, Luckstead, and Mittelhammer (2013) provide a detailed analysis of the identification and econometric issues. Applying analogous identification rationale to the European demand function (19) and São Paulo's supply relations for the United States (21) and Europe (22), we can confirm that the demand and supply parameters and the conjectural elasticities \(\theta^{su}\) and \(\theta^{se}\) can be uniquely identified and thus estimated.

Data

The data set consists of annual observations for the period 1986–2011. We collected total supply and trade data for the U.S. orange juice industry from Economic Research Service (2012a). We obtained the data for U.S. domestic orange juice supply, which is composed of Florida orange juice and other U.S. orange juice using Tables C21 and C28–C30 from the Fruit and Tree Nut Yearbook of the Economic Research Service (2012b). Total U.S. orange juice imports were disaggregated to obtain the portion of orange juice imports supplied by Brazil using U.S. orange juice import percentages by country from Food and Agriculture Organization of the United Nations (2012). The U.S. national price of orange juice was constructed using price data from Florida Department of
Citrus (2012), Bureau of Labor Statistics (2012), and Food and Agriculture Organization of the United Nations (2012). For the European market, we used Brazil’s export data, following Brown, Spreen, and Lee (2004) and Brown (2010), because Brazil is the dominant orange juice supplier in this market. Brazil’s exports and unit price were obtained from Food and Agriculture Organization of the United Nations (2012).

For input price variables in Florida’s and São Paulo’s marginal cost functions, we used producer price of oranges as the major input price, which was collected from Food and Agriculture Organization of the United Nations (2012). To account for other variable input prices, we use labor and capital price indexes for the U.S. and São Paulo fruit juice industries obtained from Becker, Gray, and Marvakov (2013) and International Labour Organization (2013). Use of several input prices to estimate the cost function leads to a multicollinearity problem because these input prices tend to move together. One approach to overcome this multicollinearity problem and yet capture the effect of all the relevant input prices is to utilize principal component analysis, which reduces the dimensionality with minimal information loss. Simply put, the principal component analysis uses eigen vectors to combine input prices into one index that captures the variance of all input prices and their effects on the cost function.

Demand shifters are income, population, and quantity of apple juice. All prices and income were converted into real terms using a GDP deflator to satisfy the homogeneity condition. Income, population, and the GDP deflators for the United States and Europe were collected from Nicita and Olarreaga (2006). In the U.S. demand function, the quantity of apple juice, collected from Economic Research Service (2012a), was included as a substitute good. In the European demand function, the quantity of apple juice, obtained from Food and Agriculture Organization of the United Nations (2012), was used as a substitute good. In the demand functions, a dummy variable—1 for 2008 and 2009 and 0 otherwise—interacts with income to account for the recent economic recession.

We collect tariff data from the World Trade Organization (2012). As per the Uruguay Round agreement, the U.S. applied specific tariff was reduced from $0.3501 per SSE (single strength equivalent) gallon in 1994 to 0.2971 per SSE gallon by 2000, and the European applied ad valorem tariff was reduced from 19% in 2004 to 15.20% by 2000. We compute transport costs from Brazil to the United States by taking the difference between the CIF and FOB fruit juice exports collected from the U.S. Census Bureau (2012). This information was used to construct the transport cost from São Paulo to Europe based on the distance between the latter two regions, which was obtained from “www.searates.com.”

Estimation

The demand functions (18) and (19) and supply relations (20)–(22) are a system of five equations with five endogenous variables ($p_u$, $p_e$, $q_{su}$, $q_{se}$, $q_f$). We estimate the parameters using nonlinear three-stage least square, which achieves consistency and increases efficiency by accounting for endogeneity in the system and cross-equation correlation in the errors. The exogenous variables in the demand and supply relation equations are used as instruments. To ensure that the objective function in the nonlinear estimation is at a global minimum, we considered a range of initial parameter values for estimating the coefficients.

Florida’s U.S. quantity supply decreased from 15,549 to 9,694 million SSE gallons from 1998 to 2011 while São Paulo’s U.S. exports remained relatively stable. Over the same period, São Paulo’s exports to the European Union declined from 12,777 to 4,812 million SSE gallons. This structural change in supply has the potential to impact the market power of both Florida and São Paulo processors. To capture the influence of this structural change on conjectural elasticities, we redefine $\theta^i$ with an intercept plus a slope times a time-varying drift variable, $B(t)$, as $\theta^i =
The negative coefficient for apple juice quantity, population, and the indicator variable for the recent recession are significant at the 7% level or better. The flexibility of demand is
\[ \theta^D = \theta^D_1 + \theta^D_2 B(t). \]
We define the drift variables to be piecewise linear:
\[ B(t) = \frac{t - t_s}{t_f - t_s} I_{(t_s,t_f)}(t) + I_{(t_f,t_N)}(t), \]
where \( t \) is time, \( t_s \) is the start and \( t_f \) is the end of the structural change, \( t_N \) is the last sample year, and \( I \) is an indicator function. The indicator function \( I_{(t_s,t_f)}(t) \) is 1 over \( t_s < t \leq t_f \) and 0 otherwise, and \( I_{(t_f,t_N)}(t) \) is 1 over \( t_f < t \leq t_N \) and 0 otherwise. Thus, the conjectural elasticities are equal to the intercept \( \theta^i = \theta^i_1 \) for \( t < t_s \), intercept plus the time weighted slope \( \theta^i = \theta^i_1 + \theta^i_2 \frac{t - t_s}{t_f - t_s} \) for \( t_s < t \leq t_f \), and intercept plus the slope \( \theta^i = \theta^i_1 + \theta^i_2 \) for \( t_f < t \leq t_N \). Based on the data presented above, we set \( t_s = 1998 \) and \( t_f = 2011 \).

Table 1 presents variable definitions. Table 2 presents the estimation results for the U.S. and European demand functions. The signs for the estimated coefficients are consistent with economic theory. In the U.S. orange juice demand equation, the estimated coefficients for the intercept, quantity, population, and the indicator variable for the recent recession are significant at the 7% level or better. The flexibility of demand is \(-0.51\) or an elasticity of \(-2.14\).\(^8\) The negative coefficient estimate for population indicates that the demand for orange juice has declined as the U.S. population has grown. The reason for this negative relationship is because of health concerns, consumption of orange juice has decreased steadily from the mid 1990s. The marginal impact of apple juice is negative, implying that orange juice and apple juice are generally complement; the rationale for this result is that consumers are tending away from orange and apple juice due to the link between obesity and high sugar consumption (Allen, 2014). The estimated coefficient for the recession indicator variable, \( D_1 \), is positive, which indicates that the recent economic crisis reduced the demand for orange juice.

In the orange juice demand equation for Europe, the estimated coefficients for the quantity variable are significant at the 1% level. The other variables included in this demand estimation are population and apple juice quantity. The negative sign of the estimated coefficient for apple juice indicates apple and orange juices are complement as in the case of the United States. The flexibility of demand is \(-1.80\) (or an elasticity of \(-0.96\)). Brown, Spreen, and Lee (2004) reports a price elasticity of demand for European orange juice at \(-0.41\), and a more recent study by Brown (2010)

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\(^8\) As noted in Tomek and Robinson (1990), the inverse of flexibility is the lower bound of the elasticity.
Table 2. U.S. and European Demand Functions

<table>
<thead>
<tr>
<th>Variable/Coefficients</th>
<th>United States ((i = u))</th>
<th>Europe ((i = e))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intercept</td>
<td>13.16 ((7.75))</td>
<td>−7.73 ((-1.21))</td>
</tr>
<tr>
<td>(q^u)</td>
<td>−0.16 ((-3.94))</td>
<td>−</td>
</tr>
<tr>
<td>(q^e)</td>
<td>−               ((-0.17))</td>
<td>−8.58 ((-1.21))</td>
</tr>
<tr>
<td>Pop(i)</td>
<td>−0.21 ((-1.93))</td>
<td>0.28 ((1.55))</td>
</tr>
<tr>
<td>(qa^i)</td>
<td>−0.12 ((-0.50))</td>
<td>−0.15 ((-0.47))</td>
</tr>
<tr>
<td>(D)</td>
<td>0.72 ((2.99))</td>
<td>−</td>
</tr>
</tbody>
</table>

Notes: t-values are in parentheses.

Table 3. Supply Relation for Florida Processors

<table>
<thead>
<tr>
<th>Variable/Coefficients</th>
<th>Florida</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intercept(t)</td>
<td>3.13 ((6.02))</td>
</tr>
<tr>
<td>(q^f)</td>
<td>0.06 ((0.59))</td>
</tr>
<tr>
<td>(lag(q^f))</td>
<td>−0.05 ((-0.94))</td>
</tr>
<tr>
<td>(pr^f)</td>
<td>1.31 ((2.92))</td>
</tr>
<tr>
<td>(\theta^f)</td>
<td>0.15 ((0.60))^a</td>
</tr>
<tr>
<td>(\theta^f)</td>
<td>0.85 ((3.47))^a</td>
</tr>
</tbody>
</table>

Notes: t-values are in parentheses and ^a is for a one-sided test.

estimates the European price elasticity of demand at −0.45 and −0.68 for ordinary least squares and instrumental variable methods, respectively.

Table 3 reports the estimated supply relation for Florida orange juice. The lagged quantity captures the year-to-year adjustment in the supply relations. The estimated coefficients for the input price is positive and significant at the 1% level. The conjectural elasticity estimate is \(\theta^f = \theta^f_a = 0.15\) for \(t < 1998\), \(\theta^f = \theta^f_a + \theta^f_b \frac{t - 1998}{2011 - 1998}\) for \(1998 < t \leq 2011\), which ranges from 0.21 to 1.00 (refer to table 5). These results imply that as concentration increases with fewer firms (as elaborated in the introduction), Florida’s processors exert more market power over time in the U.S. orange juice market.

Table 4 presents the estimated export supply relation for São Paulo orange juice to the United States and Europe. The signs for the estimated coefficients are consistent with economic theory. For exports from São Paulo to the United States, the estimated coefficients for the intercept, quantity, input prices, and transport cost are all significant at the 1% level or better. An increase in output and input prices will increase the marginal cost of production and thus orange juice price. The estimated coefficient on transport cost is positive, implying that higher shipping costs will raise the orange juice price. The conjectural elasticity estimate is \(\theta^{su} = \theta^{su}_a = 0.34\) for \(t < 1998\), \(\theta^{su} = \theta^{su}_a + \theta^{su}_b \frac{t - 1998}{2011 - 1998}\) for \(1998 < t \leq 2011\), which ranges from 0.38 to 0.93 (refer to table 5). These results indicate that the fewer number of firms in São Paulo lead to more concentration and greater market power over time in the U.S. orange juice market.

For exports from São Paulo to Europe, the estimated coefficients for the intercept, input price, and transport cost are significant at the 1% level or better. With higher quantities of output and input prices, the marginal cost of production increases, leading to an increase in the orange juice price. As in the supply relation for exports to the United States, the positive estimate for the transport cost means that an increase in the cost of shipping to Europe increases the orange juice price. The conjectural elasticity increases from 0 to 0.22, underscoring the increase in market power of São Paulo processors in the European Union.

---

9 Note that \(t_f = t_N = 2011\).
### Table 4. Supply Relations for São Paulo Processors

<table>
<thead>
<tr>
<th>Variable/Coefficients</th>
<th>Exp to U.S. ((i = su))</th>
<th>Exp to Europe ((i = se))</th>
</tr>
</thead>
<tbody>
<tr>
<td>intercept</td>
<td>(-2.53) ((-9.21))</td>
<td>(-2.53) ((-9.21))</td>
</tr>
<tr>
<td>(q^u_{su} + q^e_{se})</td>
<td>(0.23) ((3.27))</td>
<td>(0.1) ()</td>
</tr>
<tr>
<td>(p^u_{su})</td>
<td>(2.12) ((4.64))</td>
<td>(2.12) ((4.64))</td>
</tr>
<tr>
<td>(tcost)</td>
<td>(1.27) ((4.23))</td>
<td>(0.33) ((2.15))</td>
</tr>
<tr>
<td>(\theta^u_a)</td>
<td>(0.34) ((0.87)^a)</td>
<td>(0) ()</td>
</tr>
<tr>
<td>(\theta^u_b)</td>
<td>(0.59) ((2.49)^a)</td>
<td>(0.22) ((1.27)^a)</td>
</tr>
</tbody>
</table>

Notes: t-values are in parentheses and \(^a\) is for a one-sided test. The \(\) indicates a restricted parameter estimate reached its bound.

---

### Lerner Index

The Lerner Index measures the markup of price over marginal cost and is equal to the conjectural elasticity times the demand flexibility. That is, an industry’s ability to exercise oligopoly power and set price above its marginal costs depends on both the supply (conjectural elasticities) and demand (price flexibilities) conditions. Rearranging the supply relations, equations (12)–(14), the Lerner indices are expressed as

\[
\frac{p^u - \frac{\partial C^f}{\partial q^f}}{p^u} = \theta^f \varepsilon^u
\]

\[
\frac{p^u - (1 + \tau^u) \frac{\partial C^s}{\partial q^su}}{p^u} = \theta^{su} \varepsilon^u
\]

\[
\frac{p^e - (1 + \tau^e) \frac{\partial C^s}{\partial q^se}}{p^e} = \theta^{se} \varepsilon^e.
\]

The results of the Lerner Index show that in the U.S. market, Florida and São Paulo processors set their price above marginal cost. Florida processors steadily increase their markup from 5% in 1996 to 45% in 2011. Similarly, São Paulo processors also augment their markup from 12% to 42% over the same period. Thus, both Florida and São Paulo processors earn oligopolistic rents. Florida processors exert slightly greater market power in more recent years than São Paulo processors in the U.S. market because market share of the former is greater than that of the latter. For the European market, the Lerner Index shows that São Paulo exporters set prices above their marginal cost as they are able to exert market power in the European market. Since conjectural elasticity is less than 1, collusion among the exporters does not occur. However, since only three processors operate in the European market, they engage in oligopolistic competition. These results highlight the fact that both, not either, supply and demand conditions determine the ability of the processors to set price above marginal cost.

### Simulation Results

With the Doha round negotiations progressing, U.S. and European tariffs on orange juice will likely be further reduced. In this section, we analyze the effect of a 50% reduction of the U.S. and European tariffs.\(^{10}\) For the baseline simulation, we implement the existing tariff and solve the parameterized econometric model as a system of five equations (18)–(22) in five endogenous price and quantity variables (\(p^u\), \(p^e\), \(q^f\), \(q^su\), and \(q^se\)). We consider two alternate scenarios: a 50% reduction in the

\(^{10}\) We also conduct counterfactual simulation analyses for a 25% and 75% tariff reductions. In the interest of space limitation, we are not reporting the results in the paper, but these are available upon request from the authors.
### Table 5. Demand Flexibilities and Conjectural Elasticities for ASEAN

<table>
<thead>
<tr>
<th>Year</th>
<th>$\varepsilon^u$</th>
<th>$\theta^f$</th>
<th>$\theta^u$</th>
<th>$\theta^e$</th>
<th>$\theta^e^e$</th>
<th>US Market</th>
<th>Markup</th>
<th>EU Market</th>
<th>Markup</th>
</tr>
</thead>
<tbody>
<tr>
<td>1986</td>
<td>0.35</td>
<td>0.15</td>
<td>0.34</td>
<td>0.05</td>
<td>0.12</td>
<td></td>
<td></td>
<td>0.38</td>
<td>0.00</td>
</tr>
<tr>
<td>1987</td>
<td>0.35</td>
<td>0.15</td>
<td>0.34</td>
<td>0.05</td>
<td>0.12</td>
<td></td>
<td></td>
<td>0.72</td>
<td>0.00</td>
</tr>
</tbody>
</table>

$t_1 = 1998$

<table>
<thead>
<tr>
<th>Year</th>
<th>$\varepsilon^u$</th>
<th>$\theta^f$</th>
<th>$\theta^u$</th>
<th>$\theta^e$</th>
<th>$\theta^e^e$</th>
<th>US Market</th>
<th>Markup</th>
<th>EU Market</th>
<th>Markup</th>
</tr>
</thead>
<tbody>
<tr>
<td>1998</td>
<td>0.72</td>
<td>0.15</td>
<td>0.34</td>
<td>0.11</td>
<td>0.24</td>
<td></td>
<td></td>
<td>3.00</td>
<td>0.00</td>
</tr>
<tr>
<td>1999</td>
<td>0.58</td>
<td>0.21</td>
<td>0.38</td>
<td>0.12</td>
<td>0.22</td>
<td></td>
<td></td>
<td>2.55</td>
<td>0.02</td>
</tr>
<tr>
<td>2000</td>
<td>0.68</td>
<td>0.28</td>
<td>0.43</td>
<td>0.19</td>
<td>0.29</td>
<td></td>
<td></td>
<td>3.36</td>
<td>0.03</td>
</tr>
<tr>
<td>2009</td>
<td>0.48</td>
<td>0.87</td>
<td>0.84</td>
<td>0.42</td>
<td>0.40</td>
<td></td>
<td></td>
<td>0.68</td>
<td>0.19</td>
</tr>
<tr>
<td>2010</td>
<td>0.42</td>
<td>0.93</td>
<td>0.88</td>
<td>0.39</td>
<td>0.37</td>
<td></td>
<td></td>
<td>0.44</td>
<td>0.20</td>
</tr>
</tbody>
</table>

### Table 6. Impacts of Tariff Reductions, Avg. 2006–09

<table>
<thead>
<tr>
<th>Variables</th>
<th>A 50% Tariff Reduction by</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>U.S.</td>
</tr>
<tr>
<td>U.S. price $p^u$ (%)</td>
<td>−0.59</td>
</tr>
<tr>
<td>Florida’s supply $q^f$ (%)</td>
<td>−5.37</td>
</tr>
<tr>
<td>São Paulo exports to U.S. $q^{su}$ (%)</td>
<td>38.65</td>
</tr>
<tr>
<td>U.S. total quantity $q^u$ (%)</td>
<td>1.33</td>
</tr>
<tr>
<td>EU price $p^e$ (%)</td>
<td>1.73</td>
</tr>
<tr>
<td>São Paulo exports to EU $q^{se}$ (%)</td>
<td>−4.44</td>
</tr>
<tr>
<td>Florida’s market share changes in U.S.</td>
<td>−4.91%</td>
</tr>
<tr>
<td>São Paulo’s market share changes in U.S.</td>
<td>5.05%</td>
</tr>
</tbody>
</table>

U.S. tariff and a 50% reduction in the European tariff. In both of these scenarios, tariffs are 50% less in each year for the period 2009–2011. We then take the average over this period for both the baseline and alternate scenarios and compare the results for each of the alternate scenarios to those of the baseline to quantify the impacts of these two trade liberalization policies. Table 6 presents the simulation results, which are qualitatively consistent with the analytical results of the tariff analysis.

A 50% reduction in the U.S. tariff on imports from São Paulo causes São Paulo exporters to increase their exports to the United States by 38.65%. As a result of these higher exports from São Paulo, the price in the United States decreases. Because of this lower price arising from greater competition from São Paulo exporters, Florida’s producers reduce their supply by 5.37%. Consequently, São Paulo captures Florida’s market share in the United States, leading to an increase in São Paulo’s market share from 13.48% to 18.53% and a decrease in Florida’s from 75.22% to 70.31%. Since the increase in exports is more than the decline in Florida supply, the total quantity sold in the United States increases by 1.33%, which lowers the price by 0.59%. As São Paulo exports more to the United States, it diverts exports from the European Union to the United States. Consequently, São Paulo exports to the European Union fall by 4.44%, leading to a price increase of 1.73%.

A 50% decrease in the European tariff on imports from São Paulo results in a reallocation of São Paulo’s exports from the United States to Europe, leading to a 13.18% increase in exports to

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11 For the simulation we include an add factor to improve the quality of the simulation (Intriligator, 1983).

12 Note that São Paulo’s percentage increase is large because its volume of exports is small (171 million SSE gallons) relative to Florida’s sales (840 million SSE gallons). The absolute changes are very similar, with São Paulo’s export increasing by 63 million SSE gallons and Florida sales declining by 47 million SSE gallons.
Europe and a 36.42% decrease in exports to the United States. Higher European imports depress the price of orange juice by 4.78%. As São Paulo exports to the United States decline, the U.S. price rises. In response to this higher price, Florida producers expand their quantity in the United States by 5.44%. This reallocation causes Florida’s market share in the United States to increase from 75.22% to 79.77%, while São Paulo’s market share declines from 13.48% to 0.089%. Because the decline in São Paulo’s exports to the United States is more than the increase in Florida production, total quantity in the United States falls by 0.49%, which results in a modest increase of 0.20% in the U.S. price.

Next, we compare our results to those in the literature. Spreen, Brewster, and Brown (2003) developed a spatial equilibrium model with implicit supply functions to forecast the behavior of the orange juice industry. They simulated two scenarios for out-of-sample projections. In the first scenario, both U.S. and European tariffs are phased out over a fifteen-year period beginning in 2002. Their results showed minimal effects on both orange juice prices and quantities in both the United States and Europe. In the second scenario, both tariffs are eliminated in 2002. In this case, the U.S. orange juice price falls by 20%, which causes U.S. consumption to increase by 8%. The European price rises by 13%, which results in a 9% decline in consumption. Using a demand model, Brown, Spreen, and Lee (2004) examined the effect of tariff elimination on prices and found results similar to those of Spreen, Brewster, and Brown (2003).

Our simulation analysis differs from Spreen, Brewster, and Brown (2003) in four notable ways. First, they applied a dynamic spatial-equilibrium model under perfect competition, whereas our model is a static strategic trade model based on imperfect competition. Second, they estimated demand equations for the United States, Europe, and Japan and supply functions for Florida and São Paulo individually, whereas we estimate demand equations for the United States and Europe and supply relations for Florida and São Paulo orange juice processors simultaneously. Third, they used their model to project the impact of phasing out and eliminating the tariffs for an out-of-sample period, whereas we simulate the effect of a 50% tariff reduction over an in-sample period. Fourth, since they eliminate both tariffs simultaneously, their results have off-setting effects, whereas we eliminate one tariff at a time to isolate the effect of each tariff removal.

Conclusions

World orange juice production is highly concentrated in the states of Florida and São Paulo (Brazil), which produce about 85% of the total world supply. These orange juice processing states supply an average of 89% of the total U.S. market, and São Paulo processors supply about 84% of the total European market. The United States and Europe are the two largest orange juice consuming regions.

We develop a strategic trade model based on the New Trade Theory. Florida and São Paulo processors face downward-sloping demand functions. Our analytical results indicate that a reduction in the U.S. tariff causes São Paulo to reallocate its exports from Europe to the United States, which displaces orange juice supply in Florida. A tariff reduction by Europe causes São Paulo to divert its exports from the United States to Europe. Florida captures the market lost by São Paulo in the United States.

A structural econometric model, based on the strategic trade model and the new empirical industrial organization literature, is specified and estimated. The model allows the estimation of market structures ranging from perfect competition to perfect collusion/monopoly. The empirical results show that both Florida and São Paulo orange juice processors exert market power in the United States, but Florida exerts greater market power. São Paulo processors also exert modest market power in Europe.

The estimated structural model is simulated to quantify the impact of a 50% reduction in both the U.S. and European tariffs. The simulation results corroborate the qualitative results of the analytical model and also provide quantitative measures of these effects. The reduction in the U.S. tariff causes Florida’s market share to decline from 75.22% to 70.31% and São Paulo’s market share to increase
from 13.48% to 18.53% in the United States. The decline in the European tariff causes São Paulo to reallocate their exports from the United States to Europe, resulting in an increase in U.S. market share for Florida processors from 75.22% to 79.77% and a decline in the U.S. market share for São Paulo processors from 13.48% to 8.86%. Both Florida and São Paulo processors exert market power in the United States. Though the São Paulo processors have greater market power than the Florida processors in the early part of the study period, both have about 40% markup by the end of the sample.

Our analysis shows that further reduction in the U.S. tariff will increase the competition for Florida processors, but U.S. consumers will benefit. Consequently, for Florida processors to effectively compete with São Paulo processors, it is in their best interest to continue to make progress in cost-reducing technology both in orange and orange juice production.

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References


This appendix derives the comparative statics for the tariff analysis subsection. We totally differentiate the reaction functions (3)–(5) and represent them in matrix form of $Ax = d$:

$$
\begin{bmatrix}
\pi^f_{qf} & \pi^f_{qf} & 0 \\
\pi^s_{quf} & \pi^s_{quf} & \pi^s_{quf} \\
0 & \pi^s_{qve} & \pi^s_{qve}
\end{bmatrix}
\begin{bmatrix}
dq^f \\
dq^{su} \\
dq^{se}
\end{bmatrix}
= -
\begin{bmatrix}
\pi^f_{qf}d\tau^u + \pi^f_{qf}d\tau^e \\
\pi^s_{quf}d\tau^u + \pi^s_{quf}d\tau^e \\
\pi^s_{qve}d\tau^u + \pi^s_{qve}d\tau^e
\end{bmatrix}.
$$

We analyze the effect of a change in $\tau^u$ and $\tau^e$ on $q^f$, $q^{su}$, and $q^{se}$ by applying Cramer’s rule to the system (A1). First, consider the effect of changes in the tariffs on Florida orange juice supply $q^f$:

$$
dq^f = \frac{1}{|A|} \begin{vmatrix}
\pi^f_{qf} & \pi^f_{qf} & \pi^f_{qf} & 0 \\
\pi^s_{quf} & \pi^s_{quf} & \pi^s_{quf} & \pi^s_{quf} \\
0 & \pi^s_{qve} & \pi^s_{qve} & \pi^s_{qve} \\
\end{vmatrix}
\frac{d\tau^u}{|A|} \begin{vmatrix}
\pi^f_{qf} & \pi^f_{qf} & \pi^f_{qf} & 0 \\
\pi^s_{quf} & \pi^s_{quf} & \pi^s_{quf} & \pi^s_{quf} \\
0 & \pi^s_{qve} & \pi^s_{qve} & \pi^s_{qve} \\
\end{vmatrix}
\frac{d\tau^e}{|A|} > 0
$$

The impacts of changes in the tariffs on São Paulo orange juice exports to the United States $q^{su}$ are:

$$
dq^{su} = \frac{1}{|A|} \begin{vmatrix}
\pi^f_{qf} & \pi^f_{qf} & 0 \\
\pi^s_{quf} & \pi^s_{quf} & \pi^s_{quf} \\
0 & \pi^s_{qve} & \pi^s_{qve} \\
\end{vmatrix}
\frac{d\tau^u}{|A|} \begin{vmatrix}
\pi^f_{qf} & \pi^f_{qf} & 0 \\
\pi^s_{quf} & \pi^s_{quf} & \pi^s_{quf} \\
0 & \pi^s_{qve} & \pi^s_{qve} \\
\end{vmatrix}
\frac{d\tau^e}{|A|} < 0
$$

The effects of changes in the tariffs on São Paulo orange juice exports to Europe $q^{se}$ are:

$$
dq^{se} = \frac{1}{|A|} \begin{vmatrix}
\pi^f_{qf} & \pi^f_{qf} & \pi^f_{qf} & \pi^f_{qf} \\
\pi^s_{quf} & \pi^s_{quf} & \pi^s_{quf} & \pi^s_{quf} \\
0 & \pi^s_{qve} & \pi^s_{qve} & \pi^s_{qve} \\
\end{vmatrix}
\frac{d\tau^u}{|A|} \begin{vmatrix}
\pi^f_{qf} & \pi^f_{qf} & \pi^f_{qf} & \pi^f_{qf} \\
\pi^s_{quf} & \pi^s_{quf} & \pi^s_{quf} & \pi^s_{quf} \\
0 & \pi^s_{qve} & \pi^s_{qve} & \pi^s_{qve} \\
\end{vmatrix}
\frac{d\tau^e}{|A|} > 0
$$

The effects of changes in the tariffs on total quantity in the United States $q^u$ are:

$$
dq^u = dq^{su} + dq^{se} = \frac{1}{|A|} \begin{vmatrix}
\pi^s_{quf} & \pi^s_{quf} & \pi^s_{quf} \\
\pi^s_{qve} & \pi^s_{qve} & \pi^s_{qve} \\
\end{vmatrix}
\begin{vmatrix}
\pi^f_{qf} & \pi^f_{qf} & \pi^f_{qf} \\
\pi^s_{quf} & \pi^s_{quf} & \pi^s_{quf} \\
0 & \pi^s_{qve} & \pi^s_{qve} \\
\end{vmatrix}
\frac{d\tau^u}{|A|} \begin{vmatrix}
\pi^f_{qf} & \pi^f_{qf} & \pi^f_{qf} \\
\pi^s_{quf} & \pi^s_{quf} & \pi^s_{quf} \\
0 & \pi^s_{qve} & \pi^s_{qve} \\
\end{vmatrix}
\frac{d\tau^e}{|A|} .