

A Probabilistic Model of the Crop Insurance Purchase Decision

Octavio A. Ramirez and J. Scott Shonkwiler

This study proposes a probabilistic model of the individual crop insurance purchase decision that explicitly recognizes that neither the producer nor the insurer knows the exact value of the actuarially fair premium (AFP) underlying the desired policy. The model is used to explore the impact of key features of the insurer and producer AFP estimates on the probability that the producer will purchase insurance and other important indicators of program performance. The model is applied to assessing the merits of alternative premium estimation methods and to shed light on some major factors affecting the performance of the U.S. crop insurance program.

Key words: agricultural risk, comparative statics for crop insurance, crop insurance program performance, estimation of crop insurance premiums, insurance decision under uncertainty

Introduction

The U.S. crop insurance program provides important tools for farmers to manage yield and revenue risks in their operations. The Risk Management Agency (RMA), a division of the USDA, is charged with administering this program, while eighteen private insurance companies are approved to be involved in its implementation. During 2013, nearly 296 million acres were enrolled through 1.22 million individual policies, which represented about 90% of eligible crop land. The corresponding liabilities amounted to \$124 billion (U.S. Department of Agriculture, Risk Management Agency, Federal Crop Insurance Corporation, 2013).

The federal government has increasingly subsidized this program during the last two decades, and these subsidies have spurred producer participation in the various crop insurance programs. Currently, farmers as a whole pay less than 40% of the total amount of premiums required to cover all of the indemnities and administrative expenses associated with the program (U.S. Department of Agriculture, Risk Management Agency, Federal Crop Insurance Corporation, 2013). The resulting cost to tax payers during the last three crop years (2011–2013) was in excess of \$31 billion, and the government projects that an additional \$90 billion will be spent over the next ten years.

In response to this escalating burden, numerous studies over the last twenty-five years or so have analyzed and offered improvements to the actuarial performance of the federal crop insurance program (i.e., to reduce the level of government subsidies required to sustain it; or even to scrap it entirely) (Colson, Ramírez, and Fu, 2014). Some researchers have proposed alternative forms of yield insurance (e.g., Miranda, 1991; Skees, Black, and Barnett, 1997; Goodwin and Ker, 1998; Mahul, 1999; Ker and Goodwin, 2000) and revenue insurance (Gray, Richardson, and McClaskey, 1995; Hennessy, Babcock, and Hayes, 1997; Wang et al., 1998; Stokes, 2000; Coble, Heifner, and Zuniga, 2000). Others have attempted to develop methods that more accurately quantify yield and revenue risks (e.g., Gallagher, 1987; Taylor, 1990; Moss and Shonkwiler, 1993; Ramírez, 1997; Ker

Octavio A. Ramirez is a professor and the head of the Department of Agricultural and Applied Economics at the University of Georgia. J. Scott Shonkwiler is a professor in the Department of Agricultural and Applied Economics at the University of Georgia.

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and Coble, 2003; Ramírez, Misra, and Field, 2003; Norwood, Roberts, and Lusk, 2004; Ramírez and McDonald, 2006; Woodard and Sherrick, 2011; Tolhurst and Ker, 2015).

Another area of research has been devoted to understanding i) why farmers choose or choose not to participate in crop insurance programs; ii) their level of participation; and iii) their choice of instruments. Just, Calvin, and Quiggin (1999) considered data from a 1989 survey and concluded that risk aversion is a relatively weak incentive for participation. Instead, they suggested that farmers' asymmetric informational advantages lead to insuring those operations with higher expected indemnities. Their analysis assumed that yield risk was the only source of uncertainty faced by farmers and that the actuarially fair premium was treated as fixed and known by both producer and insurer. Sherrick et al. (2004) formulated a model of crop insurance participation that derived a farmer-specific reservation insurance premium. Their reservation premium depended on the expected rates of return with and without insurance and the farmer's degree of risk aversion. As would be expected under the assumption that the variance of returns under insurance is less than the variance of returns without insurance, increasing risk aversion leads to a larger reservation premium.

Subsequently, Carriquiry, Babcock, and Hart (2008) focused on the use of Actual Production History (APH) to set indemnity levels when farmers' expected yields are substantially greater than their assigned APHs. In this case, farmers will find crop insurance relatively overpriced. Upwardly biased rates can occur if yield risks grow proportionately slower than expected yields when a Loss Cost Ratio (LCR) approach is used to set rates (Woodard, Sherrick, and Schnitkey, 2011). In fact, Woodard, Sherrick, and Schnitkey (2011) found that estimated premium biases were 75–180% above actuarially fair premiums for Illinois corn. Ramírez, Carpio, and Rejesus (2011) also evaluated the bias and mean square error characteristics of various crop insurance premium estimation methods.

Most previous crop insurance studies have had an implicit or explicit assumption that the RMA-determined premium is actuarially fair (e.g., Du, Feng, and Hennessy, 2016) and that the producer knows the actuarially fair premium with certainty. However, there is no statutory requirement that the premium set at the *individual* producer level be such. At aggregate levels, premiums must reflect expected indemnities, since under the Standard Reinsurance Agreement (www.rma.usda.gov/pubs/ra) private insurance companies must retain a 20% interest in contracts designated to the Assigned Risk Fund in a state and at least a 35% interest in all other contracts designated to the Commercial Fund in each state. Further, the FCIC is mandated to an overall projected loss ratio of not greater than 1.0 (Federal Crop Insurance Act as amended through P.L. 113–79).

Pearcy and Smith (2015) noted that the catastrophic risk-loading factor increases the premium rate and provided additional ways in which the premiums established by the RMA may on average overestimate the actuarially fair premiums. Once the possibility that crop insurance premiums may be biased for some producers is introduced, then certain anomalies surrounding producer crop insurance choices may be partly explained. As pointed out by Du, Feng, and Hennessy (2016), risk-averse and even risk-neutral producers should choose subsidized coverage if premiums are actuarially fair; however, once this assumption is relaxed, failure to participate or maximize subsidy/coverage level under participation is not necessarily irrational. In this study, we go further and demonstrate that even if the premium estimates by both the insurer and the producer are unbiased, similar anomalies should be observed as long as they are subject to some level of random error (i.e., as long as the insurer and/or the producer are not certain about the actuarially fair premium).

Ramírez, Carpio, and Collart (2015) used elaborate Monte Carlo simulations to evaluate various crop insurance performance issues and assess whether federal crop insurance subsidies are equitably distributed across producers. In this study, we provide a tractable structure to those simulations using a model of the individual crop insurance purchase decision that explicitly recognizes that neither the producer nor the insurer knows the exact value of the actuarially fair premium (AFP) underlying the desired policy. Our model suggests that, due to this unavoidable uncertainty about their AFP, some

rational individuals might not purchase insurance even if they are risk averse and their premiums are highly subsidized. With additional information, this model could be used by the RMA to refine its current protocols or assess the merits of alternative premium estimation methods for improving program performance.

Our main results stem from the fact that the RMA uses an exponential adjustment to substantially shrink premium estimates toward the county mean. This creates a situation wherein the RMA estimate exhibits a substantial positive (negative) bias if the AFP is low (high) relative to the county average. Because of this characteristic of the RMA premium estimation protocol, our model predicts that high-risk farmers could on average be receiving much larger subsidies than low-risk producers, and high-risk farmers are expected to be substantially more likely than low-risk individuals to purchase insurance. In principle, we also show that alternative premium estimation methods could achieve similarly high levels of participation without the previously mentioned pitfalls and with a more equitable distribution of the subsidies across the participating producers, at a much lower overall cost to taxpayers.

Our model also suggests that, under the current protocol, further subsidy increases have much smaller impacts on program enrollment as participation expands (i.e., additional percentage gains in participation become much more costly to taxpayers). On the other hand, if subsidy levels were to be reduced, our model predicts that most farmers exiting the program would be those with lower yield risk. Since they are much better equipped to stay in business in the face of adverse growing conditions, even a substantial reduction of the subsidy rate might not have a significant detrimental effect on long-run crop acreage and supply. In contrast to past literature arguing that high subsidies are needed in part because farmers have more accurate yield risk information than the RMA, we find that increased producer knowledge about correct premiums (*ceteris paribus*) actually enhances participation.

Analytical Framework

We begin by proposing that a rational profit-maximizing producer (i) should purchase yield protection insurance if

$$(1) \quad RPP \times PPE_i > (1 - GSR) \times IPE_i,$$

where RPP is a Risk Protection Premium factor that should be greater than 1 for risk-averse producers (Mossin, 1968), PPE_i is the Producer Premium Estimate, $0 \leq GSR \leq 1$ is the Government Subsidy Rate (i.e., the percentage of the premium estimate that is subsidized by the government), and IPE_i is the Insurer Premium Estimate. Alternatively, $RPP \times PPE_i$ could be interpreted as the maximum amount that producer i is willing to pay for crop insurance.

For the purpose of our analysis, RPP and GSR are considered exogenous. However, since neither the producer nor the insurer knows the exact value of the actuarially fair premium (AFP), PPE_i and IPE_i are treated as random variables drawn from the probability distributions of the producer and the insurer estimates for the AFP. Specifically, let

$$(2) \quad \begin{aligned} PPE_i &= AFP_i + \eta_i \text{ such that } \eta_i \sim N(\mu_1, \sigma_1^2); \\ IPE_i &= AFP_i + v_i \text{ such that } v_i \sim N(\mu_2, \sigma_2^2). \end{aligned}$$

This specification allows for bias in the premium estimates when μ_1 and/or μ_2 are not equal to 0 and makes the degree of uncertainty in the estimates explicit by introducing random components with variances of σ_1^2 and σ_2^2 . The normality assumption is not unrealistic since we are not representing yields but rather the deviations of the premium estimates around the true premium. Further, there are no studies ascertaining the characteristics of the distributions of η_i (i.e., PPE_i) and v_i (i.e., IPE_i). Until such information becomes available, it seems sensible to assume normality as most other economic and econometric models do.

Producer premium estimates may be downwardly biased ($\mu_1 < 0$) because producers have lower subjective estimates of their yield variability, and insurer estimates may be upwardly biased ($\mu_2 > 0$) due to the various loads (catastrophic state and county loads; miscellaneous rate loads) that have to be imposed. Another source of insurer bias (positive or negative) may apply to producers whose average yields are substantially above or below the county reference yield due to the choice of exponent (shrinkage) for that county. Even if those estimates were unbiased (i.e., on average equal to the AFP), it stands to reason that they will be subject to some level of random error around the AFP.

It has also been argued that producers' perceptions of their fair premiums may be influenced by the actions of insurance agents (Pearcy and Smith, 2015).¹ Thus, we allow for the possibility that the producer and insurer estimates are correlated given that $Cov(PPE_i, IPE_i) = Cov(\eta_i, v_i) = \sigma_{12}$. Note also that the AFP_i is being treated as a constant in the context of a one-period setting.

Then, the probability that producer i will participate in the program is given by

$$(3) \quad \begin{aligned} Pr[RPP \times PPE_i > (1 - GSR) \times IPE_i] &= Pr[\alpha PPE_i - IPE_i > 0] \\ &= Pr[\alpha \eta_i - v_i > (1 - \alpha)AFP_i], \end{aligned}$$

where $\alpha = RPP/(1 - GSR)$. Further note that

$$(4) \quad \begin{aligned} Pr[\alpha \eta_i - v_i > (1 - \alpha)AFP_i] &= \Phi\left(\frac{(\alpha \mu_1 - \mu_2 + (\alpha - 1)AFP_i)}{(Var(\alpha \eta_i - v_i))^{1/2}}\right) \\ &= \Phi(\mu/\sigma), \end{aligned}$$

where $\mu = \alpha \mu_1 - \mu_2 + (\alpha - 1)AFP_i$, $\sigma^2 = \alpha^2 \sigma_1^2 + \sigma_2^2 - 2\alpha \rho_{12} \sigma_1 \sigma_2$, $\rho_{12} = \sigma_{12}/(\sigma_1 \sigma_2)$, and Φ denotes the standard normal cumulative distribution function. By writing the probability of participation in the crop insurance program as $\Phi((\alpha \mu_1 - \mu_2 + (\alpha - 1)AFP_i)/\sigma)$, it is easily seen that if the producer and insurer estimates are unbiased, if the producer is risk neutral, and if there is no subsidy then the chance of participation is exactly 50%.

The first question then is how the probability of participation is affected by changes in the level of bias in the producer premium estimate (μ_1), the level of bias in the insurer premium estimate (μ_2), the level of random error in the producer premium estimate (σ_1), the level of random error in the insurer premium estimate (σ_2), the level of correlation between the producer and the insurer premium estimates (ρ_{12}), the risk protection premium factor (RPP), and the rate at which the government subsidizes the estimated premiums (GSR). Thus, we are interested in the partial derivatives of $Pr[\alpha PPE_i - IPE_i > 0]$ with respect to these variables.

The following results are needed in order to obtain those derivatives (Maddala, 1983):

$$(5) \quad \partial Pr[\alpha PPE_i - IPE_i > 0]/\partial \mu = (1/\sigma)\phi(\mu/\sigma),$$

$$(6) \quad \partial Pr[\alpha PPE_i - IPE_i > 0]/\partial \sigma = -(\mu/\sigma^2)\phi(\mu/\sigma),$$

where ϕ is a standard normal density function. The first of the derivatives is

$$(7) \quad \begin{aligned} \partial Pr[\alpha PPE_i - IPE_i > 0]/\partial \mu_1 &= \{\partial Pr[\alpha PPE_i - IPE_i > 0]/\partial \mu\} \{\partial \mu/\partial \mu_1\} \\ &= \alpha(\sigma^2 2\pi)^{-1/2} \exp(-0.5(\mu/\sigma)^2). \end{aligned}$$

Since $\alpha \geq 1$, the above derivative is always positive and we conclude that, all else constant, as bias in the producer premium estimate increases so will the probability of participation. Intuitively,

¹ Percy and Smith (2015, p. 85) have noted, "Some farmers might not purchase agricultural insurance if the premium rate of agricultural insurance is too high compared to the probability of a loss or if their transaction costs are too large. Insurance agents are able to reduce these transaction costs by expending effort." Because the producer's probability of a loss and the actions of agents are unobserved, these random factors are subsumed in the random normal error.

all else constant, if the producer on average overestimates the AFP (i.e., the actual level of yield risk), then he or she should be more likely to purchase insurance. The derivative

$$(8) \quad \partial Pr[\alpha PPE_i - IPE_i > 0] / \partial \mu_2 = -(\sigma^2 2\pi)^{-1/2} \exp(-0.5(\mu/\sigma)^2)$$

is always negative. Thus, all else constant, an increasing bias in the insurer premium estimate will decrease the probability of producer participation. Naturally, the higher the premium offered, the lower the incentive for a farmer to purchase insurance. These first two results should hold regardless of the distributions assumed for the premium estimates. Further note that these first two derivatives are closely related; that is,

$$(9) \quad \partial Pr[\alpha PPE_i - IPE_i > 0] / \partial \mu_1 = -\alpha \partial Pr[\alpha PPE_i - IPE_i > 0] / \partial \mu_2.$$

Given that $\alpha \geq 1$, the absolute impact of an increased producer bias is always equal to or higher than the effect of a higher insurer bias. The next derivative we are interested in is

$$(10) \quad \begin{aligned} \partial Pr[\alpha PPE_i - IPE_i > 0] / \partial \sigma_1 &= \{ \partial Pr[\alpha PPE_i - IPE_i > 0] / \partial \sigma \} \{ (\partial \sigma / \partial \sigma^2) \partial \sigma^2 / \partial \sigma_1 \} \\ &= \{ -\mu(\sigma^4 2\pi)^{-1/2} \exp(-0.5(\mu/\sigma)^2) \} (\sigma^2)^{-1/2} (\alpha^2 \sigma_1) / 2 \\ &= -\mu(\sigma^6 2\pi)^{-1/2} \exp(-0.5(\mu/\sigma)^2) (\alpha^2 \sigma_1) / 2. \end{aligned}$$

Given that the sign of the above derivative depends on the sign of μ , the first question is the likely values for that parameter. Since $\mu = \alpha\mu_1 - \mu_2 + (\alpha - 1)AFP_i$, the condition for $\mu > 0$ is $(\alpha - 1)AFP_i > \mu_2 - \alpha\mu_1$, where $\alpha = RPP / (1 - GSR)$. Under the assumptions that $RPP \geq 1$ and the GSR is in most cases at least 0.5, these assumptions imply that $\alpha \geq 2$ and the coefficient on AFP_i would then be 1 or greater. Thus, there would have to be large and offsetting biases in the producer and insurer premium estimates for the inequality to be violated (i.e., the producer would need to substantially underestimate the fair premium ($\mu_1 \ll 0$) and/or the insurer would have to considerably over estimate it ($\mu_2 \gg 0$)). Upon assuming that $\mu > 0$, the sign of $\partial Pr[\alpha PPE_i - IPE_i > 0] / \partial \sigma_1$ is negative; therefore, an increase in σ_1 should decrease the probability of participation.

Consequently, all other factors held constant, producers who can more accurately ascertain their AFP should be more likely to purchase insurance. The intuition behind this finding can be seen by rewriting the probability of participation as $Pr[\alpha PPE_i > IPE_i]$ and graphing the probability distributions of αPPE_i and IPE_i (figure 1). In this figure, the lesser the overlap between the two distributions, the more likely that a draw from αPPE_i will exceed a draw from IPE_i , and thus the larger the probability of participation. A decrease in σ_1 (i.e., the standard deviation of PPE_i) will reduce that overlap and therefore increase the probability of participation. This figure also illustrates the larger point that, even if the individual producer-level premium estimates are unbiased (i.e., $\mu_1 = \mu_2 = 0$), some individuals might not purchase insurance even if they are risk averse and their premiums are highly subsidized (i.e., $\alpha \gg 1$) due to the unavoidable presence of random errors in those estimates (i.e., $\sigma_1 > 0$ and $\sigma_2 > 0$).

Past literature (Just, Calvin, and Quiggin, 1999) has argued that one reason why subsidies are needed to achieve higher levels of participation in the crop insurance program is that farmers have more information and hence more certainty than the insurer about the actual AFP. In contrast, the above result suggests that enhanced producer knowledge is beneficial to participation. Next we are interested on the impact of the level of accuracy in the insurer estimate on participation:

$$(11) \quad \begin{aligned} \partial Pr[\alpha PPE_i - IPE_i > 0] / \partial \sigma_2 &= \{ \partial Pr[\alpha PPE_i - IPE_i > 0] / \partial \sigma \} \{ (\partial \sigma / \partial \sigma^2) \partial \sigma^2 / \partial \sigma_2 \} \\ &= \{ -\mu(\sigma^4 2\pi)^{-1/2} \exp(-0.5(\mu/\sigma)^2) \} (\sigma^2)^{-1/2} (\sigma_2) / 2 \\ &= -\mu(\sigma^6 2\pi)^{-1/2} \exp(-0.5(\mu/\sigma)^2) (\sigma_2) / 2. \end{aligned}$$

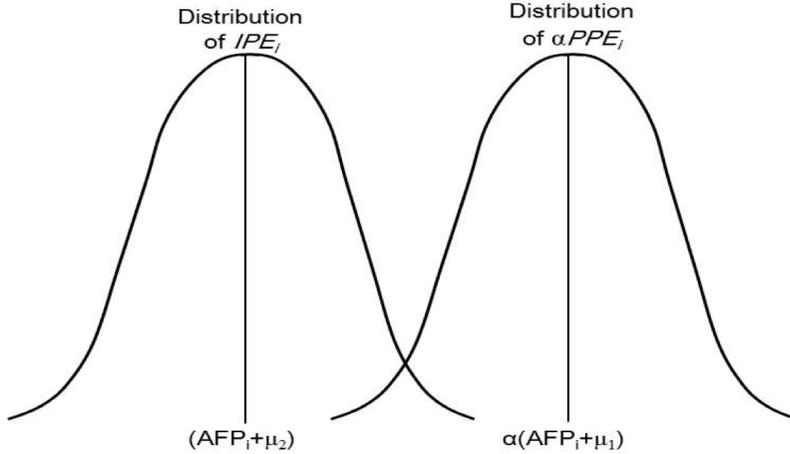


Figure 1. Distribution of IPE_i versus αPPE_i

Since $-\mu(\sigma^6 2\pi)^{-1/2} \exp(-0.5(\mu/\sigma)^2)(\sigma_2)/2$ is always opposite the sign of μ and μ is believed to be positive, $\partial Pr[\alpha PPE_i - IPE_i > 0]/\partial \sigma_2$ should be negative. The intuition behind this result is similar to that of the previous derivative with respect to σ_1 . From figure 1, a decrease in the variability of IPE_i around its mean $(AFP_i + \mu_2)$ increases the probability that $\alpha PPE_i > IPE_i$ as long as $\mu > 0$ (i.e., as long as the distribution of αPPE_i is to the right of the distribution of IPE_i). Figure 1 also shows that these variance results should hold even if the probability distributions of the premiums are kurtotic or asymmetric in shape. Holding everything else constant, a decrease in variance would always shift the upper tail of the distribution of IPE_i to the left or the lower tail of the distribution of αPPE_i to the right (i.e., diminish the overlap between the two distributions).

Since $\{\partial Pr[\alpha PPE_i - IPE_i > 0]/\partial \sigma_1\} = \{\partial Pr[\alpha PPE_i - IPE_i > 0]/\partial \sigma_2\} \alpha^2 \sigma_1/\sigma_2$ and given that $\alpha > 1$ (in fact, if $GSR > 0.50$, then $\alpha > 2$), a change in producer error has a much larger impact on the probability of participation than a change in insurer error if σ_1 and σ_2 are similar in magnitude. Thus, educational programs aimed at improving farmers’ capacity to assess their yield risks could have a substantial effect on increasing participation in crop insurance.

Another derivative of interest is

$$\begin{aligned}
 \partial Pr[\alpha PPE_i - IPE_i > 0]/\partial \rho_{12} &= \{\partial Pr[\alpha PPE_i - IPE_i > 0]/\partial \sigma\} \{(\partial \sigma/\partial \sigma^2) \partial \sigma^2/\partial \rho_{12}\} \\
 (12) \qquad \qquad \qquad &= \{-\mu(\sigma^4 2\pi)^{-1/2} \exp(-0.5(\mu/\sigma)^2)\}(\sigma^2)^{-1/2}(-\alpha \sigma_1 \sigma_2)/2 \\
 &= \mu(\sigma^6 2\pi)^{-1/2} \exp(-0.5(\mu/\sigma)^2) \alpha \sigma_1 \sigma_2/2.
 \end{aligned}$$

Clearly, since $\alpha > 1$, $\sigma_1 > 0$, $\sigma_2 > 0$, and μ is believed to be positive, $\partial Pr[\alpha PPE_i - IPE_i > 0]/\partial \rho_{12} > 0$ and an increase in ρ_{12} is deemed to improve the probability of participation. Obviously, the same holds when the derivative is taken with respect to σ_{12} . It is economically intuitive that a higher correlation between the producer and the insurer premium estimate would facilitate a “meeting of the minds” and make it more likely for a producer to purchase crop insurance.

The statistical intuition behind this result can also be surmised from figure 1. If αPPE_i is below its mean $\alpha(AFP_i + \mu_1)$, a higher correlation would make it more likely for IPE_i to be below its mean $(AFP_i + \mu_2)$ as well, which increases the probability that $\alpha PPE_i > IPE_i$. Again this result should hold under kurtotic and asymmetric unimodal distributions as well. An empirical implication of this finding is that educational programs that enhance the producers’ trust in the accuracy of the RMA premium quotes would increase this correlation and thus heighten demand for crop insurance.

Perhaps the most important derivative is the one with respect to the GSR (i.e., the rate at which the insurer premium estimates are subsidized to arrive to the final premium quote presented to the

producer). This is given by

$$\begin{aligned}
 \partial Pr[\alpha PPE_i - IPE_i > 0] / \partial GSR &= \{ \partial Pr[\alpha PPE_i - IPE_i > 0] / \partial \mu \} \{ \partial \mu / \partial GSR \} \\
 &+ \{ \partial Pr[\alpha PPE_i - IPE_i > 0] / \partial \sigma \} \{ (\partial \sigma / \partial \sigma^2) \partial \sigma^2 / \partial GSR \} \\
 &= \{ \partial Pr[\alpha PPE_i - IPE_i > 0] / \partial \mu \} \{ \alpha(\mu_1 + AFP_i) / (1 - GSR) \} \\
 (13) \quad &+ \{ \partial Pr[\alpha PPE_i - IPE_i > 0] / \partial \sigma \} \{ (\sigma^2)^{-1/2} (\alpha^2 \sigma_1^2 - \alpha \rho_{12} \sigma_1 \sigma_2) / 2(1 - GSR) \} \\
 &= \alpha(\mu_1 + AFP_i) (\sigma^2 2\pi)^{-1/2} \exp(-0.5(\mu/\sigma)^2) / (1 - GSR) \\
 &- \mu (\sigma^6 2\pi)^{-1/2} \exp(-0.5(\mu/\sigma)^2) (\sigma^2 - \sigma_2^2) / 2(1 - GSR).
 \end{aligned}$$

Intuitively, from figure 1, the first term captures the fact that an increase in the GSR increases α , which raises the mean of αPPE_i , which unequivocally boosts the probability of participation ($Pr[\alpha PPE_i > IPE_i]$). As expected, this “mean” effect of a change in the GSR on $Pr[\alpha PPE_i > IPE_i]$ is always positive. The second term captures the fact that an increase in the GSR also raises the variance of αPPE_i , which would decrease $Pr[\alpha PPE_i > IPE_i]$ as long as $E[\alpha PPE_i] > E[IPE_i]$ (i.e., $\mu > 0$). This opens the possibility for $\partial Pr[\alpha PPE_i - IPE_i > 0] / \partial GSR < 0$. Considering both the mean and the variance effect, the sign of $\partial Pr[\alpha PPE_i - IPE_i > 0] / \partial GSR$ squarely depends on whether

$$\begin{aligned}
 (14) \quad &\alpha(\mu_1 + AFP_i) - (\alpha\mu_1 - \mu_2 + (\alpha - 1)AFP_i)(\sigma^2 - \sigma_2^2) / 2\sigma^2 > 0 \\
 &\text{or } (\alpha\mu_1 + \mu_2 + (\alpha + 1)AFP_i + \mu\sigma_2^2 / \sigma^2) / 2 > 0.
 \end{aligned}$$

While possible, the conditions under which a negative variance effect could offset the positive mean effect and cause an increase in the GSR to reduce the probability of participation are unlikely. We therefore maintain that the derivative of the probability of participation with respect to the GSR is generally positive, which is what a casual observer with a basic economic intuition would expect. The last derivative that we are interested in is

$$\begin{aligned}
 \partial Pr[\alpha PPE_i - IPE_i > 0] / \partial RPP &= \{ \partial Pr[\alpha PPE_i - IPE_i > 0] / \partial \mu \} \{ \partial \mu / \partial RPP \} \\
 &+ \{ \partial Pr[\alpha PPE_i - IPE_i > 0] / \partial \sigma \} \{ (\partial \sigma / \partial \sigma^2) \partial \sigma^2 / \partial RPP \} \\
 &= (\mu_1 + AFP_i) (\sigma^2 2\pi)^{-1/2} \exp(-0.5(\mu/\sigma)^2) / (1 - GSR) \\
 (15) \quad &- \mu (\sigma^6 2\pi)^{-1/2} \exp(-0.5(\mu/\sigma)^2) (\alpha\sigma_1^2 - \rho_{12}\sigma_1\sigma_2) / 2(1 - GSR) \\
 &= (\mu_1 + AFP_i) (\sigma^2 2\pi)^{-1/2} \exp(-0.5(\mu/\sigma)^2) / (1 - GSR) \\
 &- \mu (\sigma^6 2\pi)^{-1/2} \exp(-0.5(\mu/\sigma)^2) ((\sigma^2 - \sigma_2^2) / \alpha) / 2(1 - GSR).
 \end{aligned}$$

Notably, for $\partial Pr[\alpha PPE_i - IPE_i > 0] / \partial RPP > 0$, $(\mu_1 + AFP_i) - (\mu/\alpha)(\sigma^2 - \sigma_2^2) / 2\sigma^2 > 0$, or $(\alpha\mu_1 + \mu_2 + (\alpha + 1)AFP_i + \mu\sigma_2^2 / \sigma^2) / 2\alpha > 0$, which is proportional to the same condition established above for $\partial Pr[\alpha PPE_i - IPE_i > 0] / \partial GSR$ to be positive. Thus, as expected, it is most likely that an increase in the risk protection premium will have a positive effect on participation.

For the purposes of this study we are particularly interested in the expected value ($E[.]$) and the variance ($Var[.]$) of the effective per acre subsidy (PAS) received by a participating producer relative to his/her AFP. These are defined as

$$(16) \quad PAS_p = AFP_p - (1 - GSR)IPE_p,$$

$$(17) \quad E[PAS_p] = AFP_p - (1 - GSR)E[IPE_p], \text{ and}$$

$$(18) \quad Var[PAS_p] = (1 - GSR)^2 Var[IPE_p],$$

where the subscript p indicates that we are interested in the distribution of the insurer premium estimate (IPE) and the subsidy (PAS) only among the producers who participate in the program.

Since participation is not universal, we have, in general, $E[IPE_p] \neq E[IPE]$ and $Var[IPE_p] \neq Var[IPE]$. The first task then is to ascertain how $E[IPE_p]$ and $Var[IPE_p]$ are related to $E[IPE] = AFP + \mu_2$ and $Var[IPE] = (\sigma_2)^2$. Using Greene (2012, p. 836) regarding the mean and the variance of a normally distributed random variable (IPE) contingent on the realization of a binomial random variable (whether or not the producer participates), these relationships can be expressed as follows:

$$(19) \quad E[IPE_p] = E[IPE] + \delta\sigma_2\omega = AFP + \mu_2 + \delta\sigma_2\omega,$$

and

$$(20) \quad Var[IPE_p] = Var[IPE](1 - \delta^2\omega(\omega + \mu/\sigma)) = (\sigma_2)^2(1 - \delta^2\omega(\omega + \mu/\sigma)),$$

where α , μ , and σ are as previously defined; $\delta = (\alpha\rho_{12}\sigma_1\sigma_2 - \sigma_2^2)/\sigma_2\sigma$; $\omega = \varphi(\mu/\sigma)/\Phi(\mu/\sigma)$; and φ and Φ are the standard normal pdf and cdf respectively.

First note that if $\sigma_2 = 0$ (i.e., if there is no uncertainty in the insurer premium estimates), then $E[IPE_p] = AFP + \mu_2 = E[IPE]$. Alternatively, if $\rho_{12} = 0$ (i.e., there is no correlation between the producer and the insurer premium estimates), then δ is unambiguously negative and—since σ_2 and ω are always positive— $E[IPE_p] < E[IPE]$. In this case the premiums paid by participants will, on average, be less than those quoted to all producers. However, if $\rho_{12} > 0$, the sign of δ depends on $\alpha\rho_{12}\sigma_1\sigma_2 - (\sigma_2)^2$. That is, if $\sigma_1/\sigma_2 < 1/\alpha\rho_{12}$, $E[IPE_p] < E[IPE]$, and vice versa. As an example, if $\alpha = 2$ and $\rho_{12} = 0.50$, $\sigma_1/\sigma_2 < 1/\alpha\rho_{12}$ as long as $\sigma_1/\sigma_2 < 1$. Then, if σ_1 is substantially larger (smaller) than σ_2 it is more likely that the average of the insurer premium estimates for the participating producers is higher (lower) than the average of the premium estimates for all producers. Also note that larger values of α and/or ρ_{12} would require lower values of σ_1 relative to σ_2 for this inequality to hold.

Regarding $Var[IPE_p]$, the term $\omega(\omega + \mu/\sigma)$ is bounded between 0 and 1, and it can be shown that δ is confined within this range as well (Greene, 2012). Therefore, $(1 - \delta^2\omega(\omega + \mu/\sigma)) \leq 1$ and $Var[IPE_p] \leq \sigma_2^2$, which means that the variance of insurer premium estimates for participating producers is always lower than the variance of estimates for all producers.

These two formulas allow us to compute the expected value and the variance of the per acre subsidy received by participating producers based on the values of μ_2 and $(\sigma_2)^2$ (i.e., on the bias and variance of the insurer premium estimates for all producers). Also note that the bias and the variance of the producer premium estimate (μ_1 and $(\sigma_1)^2$) as well as the level of correlation between the two estimates (ρ_{12}) indirectly affect both $E[PAS_p]$ and $Var[PAS_p]$ through the correction factors $\{\delta\sigma_2\omega$ and $(1 - \delta^2\omega(\omega + \mu/\sigma))\}$ that have to be applied to $E[IPE]$ and $Var[IPE]$ in order to obtain $E[IPE_p]$ and $Var[IPE_p]$. Because of those correction factors, it is possible that higher $E[IPE]$ and $Var[IPE]$ (i.e., μ_2 and $(\sigma_2)^2$) are actually associated with lower $E[IPE_p]$ and $Var[IPE_p]$. Also note that while the GSR and RPP do not affect $E[IPE]$ and $Var[IPE]$, they do impact $E[IPE_p]$ and $Var[IPE_p]$ through those factors.

A final measure of interest is the percentage of indemnities to be paid to a participating producer that is expected to be funded by the government (PFG). Note that $(1 - GSR)E[IPE_p]$ computes the expected contribution by the participating producer, while AFP_p is the expected indemnity to be paid to that producer. Thus, the $E[PFG_p]$ is calculated as follows:

$$(21) \quad E[PFG_p] = 1 - (1 - GSR)E[IPE_p]/AFP_p.$$

The above formulas can be used to compute the probability of participation (POP), the expected value and the variance of the per acre subsidy accruing to each participating producer, and the proportion of the resulting indemnities to be funded by the government (PFG) for any given combination of RPP, GSR, μ_1 , μ_2 , σ_1 , σ_2 , and ρ_{12} .

Using the case of dry-land corn in the state of Illinois as an example, the following section illustrates some potential applications of our theoretical results.

Potential Applications

We discuss a potential application to the case of Iroquois County, which is one of the top corn-producing counties in Illinois. First we illustrate the potential for relatively large differences between the actual RMA premium estimates and the corresponding AFPs. Then we use the formulas derived in the previous section to compute the probability of participation (*POP*) and the expected value and variance of the per acre subsidy ($E[PAS_p]$ and $Var[PAS_p]$) accruing to a set of hypothetical producers under several likely scenarios and discuss how these can vary widely depending on the underlying AFPs. The elasticities of the probability of participation with respect to changes in the characteristics of the insurer and producer premium estimates (i.e., μ_1 , σ_1 , μ_2 , σ_2 , and ρ_{12}) and other key variables such as the GSR are computed and discussed as well. Finally, we illustrate how our model could be used by the RMA to assess the merits of alternative premium estimation methods for potentially improving program performance.

Table 1 contains key crop insurance statistics for Iroquois County. Specifically, in the first three columns, we present the AFPs corresponding to various mean-standard deviation (M-S) combinations, assuming normally distributed yields and a 75% coverage level (CL). The M-S combinations included in the analyses were selected to be centered at the values (155 and 39.7 bu/acre) for which the corresponding AFP (17.31/acre) is equal to the RMA premium estimate for an Actual Production History (APH) yield of 155 bu/acre (bolded row in table 1).² The independent M-S combination design was based on Goodwin (1994), who only found weak and mixed evidence of mean-variance correlation in a variety of crops including dryland and irrigated corn. There is considerable variation in the AFPs associated with the selected M-S combinations. As expected, the lowest AFP (\$6.7/acre) corresponds to the highest mean (175 bu/acre) and the lowest standard deviation (32.5 bu/acre) scenario, and the highest AFP (\$33.3/acre) corresponds to the lowest mean (135 bu/acre) and the highest standard deviation (47.5 bu/acre) scenario. If the M-S combinations are constrained to ranges from 145–165 and 35–45 bu/acre, respectively, the AFPs span from \$10.26/acre to \$26.78/acre.

The average premium estimates presented in table 1 were obtained from the official RMA crop insurance cost estimator (<https://ewebapp.rma.usda.gov/apps/costestimator/Estimates/DetailedEstimate.aspx>, accessed June 2014). In order to compute a premium quote, this estimator only requires the APH of the farm to be insured, which is usually calculated as a simple average of the last ten yield realizations. For the purpose of our analyses, the objective is to retrieve the expected RMA premium estimate corresponding to a farm unit with the assumed yield distribution mean and standard deviation. The process for accomplishing this objective entailed repeatedly drawing random samples (100 samples of size $n = 10$) from the underlying yield distribution, calculating the APH from each sample, using the insurance cost estimator to obtain a premium quote given that APH, and then computing the average and standard deviation of the resulting quotes.

Notably, the individual premium quotes as well as their averages (table 1, column 4) are almost the same regardless of the yield M-S combination underlying the APH values entered into the estimator. In other words, the quotes received by corn producers in Iroquois County are expected to be within the very narrow range of \$17.31–\$17.42/acre, regardless of the characteristics (i.e., the mean and standard deviation) of the underlying yield distributions. In contrast, as noted earlier, their AFPs could range from as little as \$6.7 to as much as \$33.3/acre. As a result, the premium estimate for a farmer with high average yields and low downside yield volatility could be more than twice what is actuarially fair, while the premium quote for a producer with low average yields and

² The APH is the only farm-level yield statistic used by the RMA to compute the premium estimates. Choices of yields and their variances were based on examination of recent Iroquois County data.

Table 1. Actuarially Fair Premiums (AFP), Average RMA Premium Estimates (RMAEST), and Average Premiums Quoted to Producers (PROQUO) Corresponding to Yield Distributions with Various Mean (MEAN) and Standard Deviation (STD) Combinations

| MEAN | STD | AFP | RMAEST | PROQUO |
|--------------|-------------|--------------|--------------|-------------|
| 135.0 | 32.5 | 12.58 | 17.42 | 7.84 |
| 145.0 | 32.5 | 10.82 | 17.24 | 7.76 |
| 155.0 | 32.5 | 9.26 | 17.31 | 7.79 |
| 165.0 | 32.5 | 7.9 | 17.42 | 7.84 |
| 175.0 | 32.5 | 6.7 | 17.41 | 7.83 |
| 135.0 | 35.0 | 15.6 | 17.42 | 7.84 |
| 145.0 | 35.0 | 13.62 | 17.24 | 7.76 |
| 155.0 | 35.0 | 11.84 | 17.31 | 7.79 |
| 165.0 | 35.0 | 10.26 | 17.42 | 7.84 |
| 175.0 | 35.0 | 8.85 | 17.41 | 7.83 |
| 135.0 | 37.5 | 18.83 | 17.42 | 7.84 |
| 145.0 | 37.5 | 16.64 | 17.24 | 7.76 |
| 155.0 | 37.5 | 14.66 | 17.31 | 7.79 |
| 165.0 | 37.5 | 12.87 | 17.42 | 7.84 |
| 175.0 | 37.5 | 11.26 | 17.41 | 7.83 |
| 135.0 | 39.7 | 21.82 | 17.42 | 7.84 |
| 145.0 | 39.7 | 19.46 | 17.24 | 7.76 |
| 155.0 | 39.7 | 17.31 | 17.31 | 7.79 |
| 165.0 | 39.7 | 15.34 | 17.42 | 7.84 |
| 175.0 | 39.7 | 13.57 | 17.41 | 7.83 |
| 135.0 | 42.5 | 25.81 | 17.42 | 7.84 |
| 145.0 | 42.5 | 23.25 | 17.24 | 7.76 |
| 155.0 | 42.5 | 20.89 | 17.31 | 7.79 |
| 165.0 | 42.5 | 18.72 | 17.42 | 7.84 |
| 175.0 | 42.5 | 16.74 | 17.41 | 7.83 |
| 135.0 | 45.0 | 29.51 | 17.42 | 7.84 |
| 145.0 | 45.0 | 26.78 | 17.24 | 7.76 |
| 155.0 | 45.0 | 24.25 | 17.31 | 7.79 |
| 165.0 | 45.0 | 21.92 | 17.42 | 7.84 |
| 175.0 | 45.0 | 19.76 | 17.41 | 7.83 |
| 135.0 | 47.5 | 33.33 | 17.42 | 7.84 |
| 145.0 | 47.5 | 30.45 | 17.24 | 7.76 |
| 155.0 | 47.5 | 27.76 | 17.31 | 7.79 |
| 165.0 | 47.5 | 25.26 | 17.42 | 7.84 |
| 175.0 | 47.5 | 22.95 | 17.41 | 7.83 |

high downside yield volatility could be about half of what is actuarially fair. In the context of our theoretical model, this means that the distribution of the RMA premium estimates for producer i can exhibit a very high positive or negative bias (μ_2) depending on whether that individual’s yield risk is low or high, but the distribution has an extremely low variance ($(\sigma_2)^2$).

The next step in our analysis is to use the information presented in table 1 and the formulas derived in the previous section to compute the probability of participation (POP) and the expected value and variance of the per acre subsidy ($E[PAS_p]$ and $Var[PAS_p]$) accruing to a set of hypothetical producers with AFPs spanning the previously discussed range. Specifically, in the baseline scenario, we assume that farmers are willing to pay a 10% risk protection premium (i.e., $RPP = 1.10$) for the 75% coverage level (CL), the government subsidizes 55% of the estimated premium payments

Table 2. Select Statistics for the First and Second Scenarios (All Labels Defined in the Text)

| FREQ | First Scenario | | | | | Second Scenario | | | | | | |
|-------|----------------|-------|------------|--------------|--------|---------------------|--------------|-------|------------|--------------|-------|---------------------|
| | AFP | POP | $E[PAS_p]$ | $S[PAS_p]$ | PFG | ϵ_{popGSR} | AFP | POP | $E[PAS_p]$ | $S[PAS_p]$ | PFG | ϵ_{popGSR} |
| 0.025 | 7.000 | 0.481 | -0.789 | 0.023 | -0.113 | 4.093 | 7.000 | 0.953 | 2.331 | 1.558 | 0.333 | 0.157 |
| 0.028 | 8.000 | 0.677 | 0.211 | 0.023 | 0.026 | 2.295 | 8.000 | 0.953 | 2.664 | 1.780 | 0.333 | 0.157 |
| 0.032 | 9.000 | 0.803 | 1.211 | 0.023 | 0.135 | 1.328 | 9.000 | 0.953 | 2.997 | 2.003 | 0.333 | 0.157 |
| 0.035 | 10.000 | 0.878 | 2.211 | 0.023 | 0.221 | 0.795 | 10.000 | 0.953 | 3.330 | 2.225 | 0.333 | 0.157 |
| 0.039 | 11.000 | 0.923 | 3.211 | 0.023 | 0.292 | 0.493 | 11.000 | 0.953 | 3.663 | 2.448 | 0.333 | 0.157 |
| 0.043 | 12.000 | 0.949 | 4.211 | 0.023 | 0.351 | 0.316 | 12.000 | 0.953 | 3.996 | 2.670 | 0.333 | 0.157 |
| 0.046 | 13.000 | 0.966 | 5.211 | 0.023 | 0.401 | 0.210 | 13.000 | 0.953 | 4.329 | 2.893 | 0.333 | 0.157 |
| 0.050 | 14.000 | 0.976 | 6.211 | 0.023 | 0.444 | 0.143 | 14.000 | 0.953 | 4.662 | 3.116 | 0.333 | 0.157 |
| 0.054 | 15.000 | 0.983 | 7.211 | 0.023 | 0.481 | 0.101 | 15.000 | 0.953 | 4.995 | 3.338 | 0.333 | 0.157 |
| 0.057 | 16.000 | 0.987 | 8.211 | 0.023 | 0.513 | 0.073 | 16.000 | 0.953 | 5.328 | 3.561 | 0.333 | 0.157 |
| 0.061 | 17.000 | 0.990 | 9.211 | 0.023 | 0.542 | 0.054 | 17.000 | 0.953 | 5.661 | 3.783 | 0.333 | 0.157 |
| 0.061 | 18.000 | 0.992 | 10.211 | 0.023 | 0.567 | 0.041 | 18.000 | 0.953 | 5.994 | 4.006 | 0.333 | 0.157 |
| 0.057 | 19.000 | 0.994 | 11.211 | 0.023 | 0.590 | 0.031 | 19.000 | 0.953 | 6.327 | 4.228 | 0.333 | 0.157 |
| 0.054 | 20.000 | 0.995 | 12.211 | 0.023 | 0.611 | 0.025 | 20.000 | 0.953 | 6.660 | 4.451 | 0.333 | 0.157 |
| 0.050 | 21.000 | 0.996 | 13.211 | 0.023 | 0.629 | 0.020 | 21.000 | 0.953 | 6.993 | 4.673 | 0.333 | 0.157 |
| 0.046 | 22.000 | 0.997 | 14.211 | 0.023 | 0.646 | 0.016 | 22.000 | 0.953 | 7.326 | 4.896 | 0.333 | 0.157 |
| 0.043 | 23.000 | 0.997 | 15.211 | 0.023 | 0.661 | 0.013 | 23.000 | 0.953 | 7.659 | 5.118 | 0.333 | 0.157 |
| 0.039 | 24.000 | 0.998 | 16.211 | 0.023 | 0.675 | 0.011 | 24.000 | 0.953 | 7.992 | 5.341 | 0.333 | 0.157 |
| 0.035 | 25.000 | 0.998 | 17.211 | 0.023 | 0.688 | 0.009 | 25.000 | 0.953 | 8.325 | 5.563 | 0.333 | 0.157 |
| 0.032 | 26.000 | 0.998 | 18.211 | 0.023 | 0.700 | 0.008 | 26.000 | 0.953 | 8.658 | 5.786 | 0.333 | 0.157 |
| 0.028 | 27.000 | 0.998 | 19.211 | 0.023 | 0.712 | 0.007 | 27.000 | 0.953 | 8.991 | 6.009 | 0.333 | 0.157 |
| 0.025 | 28.000 | 0.999 | 20.211 | 0.023 | 0.722 | 0.006 | 28.000 | 0.953 | 9.324 | 6.231 | 0.333 | 0.157 |
| 0.021 | 29.000 | 0.999 | 21.211 | 0.023 | 0.731 | 0.005 | 29.000 | 0.953 | 9.657 | 6.454 | 0.333 | 0.157 |
| 0.017 | 30.000 | 0.999 | 22.211 | 0.023 | 0.740 | 0.004 | 30.000 | 0.953 | 9.990 | 6.676 | 0.333 | 0.157 |
| 0.014 | 31.000 | 0.999 | 23.211 | 0.023 | 0.749 | 0.004 | 31.000 | 0.953 | 10.323 | 6.899 | 0.333 | 0.157 |
| 0.010 | 32.000 | 0.999 | 24.211 | 0.023 | 0.757 | 0.003 | 32.000 | 0.953 | 10.656 | 7.121 | 0.333 | 0.157 |
| | PPR = | 0.955 | | PFG = | 0.506 | 0.219 | PPR = | 0.953 | | PFG = | 0.317 | 0.150 |

(GSR=0.55, which is the subsidy rate currently applied by the RMA for that coverage level), the producer premium estimate is unbiased ($\mu_1 = 0$), the standard deviation of the producer premium estimate is 25% of the AFP ($\sigma_1 = AFP/4$), the expected value of the insurer premium estimate is \$17.31 (i.e., $\mu_2 = 17.31 - AFP$), and the standard deviation of the insurer premium estimate is very small ($\sigma_2 = 0.05$). In light of the minimal and seemingly random variability in the insurer quote, it is assumed that the linear correlation between the two premium estimates is 0 ($\rho_{12} = 0$).

A 10% risk protection premium implies that the producer is willing to pay 10% more for insurance than a risk-neutral individual. While it is not possible to compare this to a risk-aversion coefficient, our thinking is that it represents a normal, moderate level of risk aversion. It is plausible that $\mu_1 = 0$, which assumes that the producer premium estimate is unbiased, even though there is no empirical evidence for or against this assumption. Additionally $\sigma_1 = AFP/4$ sets the standard deviation of the distribution of the producer premium estimate at one-fourth of the corresponding AFP, which again is merely a sensible assumption. Given that the RMA premium estimates were found to be tightly centered around \$17.31/acre (table 1, column 4) the RMA bias (μ_2) is approximately $17.31 - AFP$, and $\sigma_2 = 0.05$ again reflects the fact that all the RMA premium estimates are tightly bound around \$17.31/acre, which means that their distribution has a very small variance.

The statistics corresponding to this first scenario are presented in the first seven columns of table 2. First note that despite the high (55%) premium subsidy rate, according to equation (4), a producer with a very low AFP (\$7/acre) would only have a 48.1% probability of participation. Using equation (17), we calculate that if a producer purchases insurance, the expected per acre subsidy would be -\$0.79. In other words, this farmer would on average experience a \$0.79 annual per acre loss by participating in the program. Also note that due to the very small level of variability in the insurer premium estimate, the standard deviation associated with this subsidy value (computed equation 18) is negligible (\$0.023 per acre). Finally, according to equation (21), the expected *PF*G is -0.113, which means that such a producer would on average pay 11.3% more than what is needed in order to cover his or her expected indemnities.

As previously indicated, if the M-S combinations are constrained to ranges from 145–165 and 35–45 bu/acre, respectively, then the AFPs lie between \$10.26/acre and \$26.78/acre. A producer with an AFP of \$10/acre would exhibit an 87.8% probability of participation and—conditional on participation—expect a positive \$2.21/acre subsidy with a high level of certainty (standard deviation of \$0.023/acre). In this case, the government would have to fund 22.1% of the expected indemnities. In contrast, a farmer at the other extreme of this narrower range (AFP of \$27/acre) would show a nearly 100% probability of participation, expect a \$19.21/acre subsidy with a similarly high degree of certainty, and the government would need to pay for 71.2% of the indemnities to be claimed by this producer (table 2).

The formulas provided in the previous section can also be used to compute the elasticities of the probability of participation with respect to changes in the characteristics of the insurer and producer premium estimates (i.e., μ_1 , σ_1 , μ_2 , σ_2 , and ρ_{12}) and other key variables such as the GSR. For illustrative purposes, the elasticities with respect to the GSR ($\epsilon_{pop,GSR}$) are presented in table 2 as well. These elasticities are much larger at the lower AFPs, decline quickly, and become very small after the *POP*s reach 99%. At AFP=\$7/acre (*POP* = 0.481), a 1% raise in the GSR would increase the *POP* by 4.09%. However, at an AFP of \$10/acre (*POP* = 0.878), the elasticity declines to 0.795%, and when APF=14 (*POP* = 0.976), it is just 0.143%. This suggests that proportional increases in the GSR will have much smaller relative impacts on program enrollment as overall participation expands (i.e., as the producer participation rate increases, additional percentage gains in participation will be much more costly to taxpayers).

In order to aggregate over the individual impacts, we need to assume a frequency distribution for the AFPs (i.e., specify the percentages of the producer population that likely exhibit different AFP values ranging from \$7/acre to \$32/acre in increments of \$1/acre). Our assigned frequencies (column 1, labeled *FREQ*, in table 2) correspond to a right-truncated triangular distribution centered

at \$17.5/acre. The assumption of a right-truncated distribution is based on the mean and range of the AFP values reported in table 1 (column 3), which correspond to the assumed mean-variance combinations for the underlying yield distributions. We could assume other distributions, which would certainly affect the results. Under this frequency assignment, approximately 12% of the producers in the population exhibit AFPs less than \$11/acre, while 14% exhibit AFPs of more than \$25/acre and about 50% are assumed to have AFPs within the tight range of \$14–\$21/acre.

The proportion of this hypothetical population of farmers that is expected to purchase insurance can then be computed by the weighted average of the individual *POPs* (i.e., the sum of the product of column 1 and column 3). The resulting aggregate producer participation rate (*PPR*) is 95.5%, which seems reasonable for a 75% coverage level given the 55% premium subsidy.³ By extension, the expected overall percentage of the indemnities that would have to be funded by the government can be computed by the sum of the individual *PFGs* (column 6) weighted by columns 1 and 3. The resulting aggregate *PFG* for this particular baseline scenario is 50.6%.

Aggregate elasticities can be analogously calculated. Notably, our model predicts that a 1% reduction in the GSR would decrease the overall *PPR* by just 0.219%, but it would lower the expected proportion of the total indemnities to be paid by the government (*PFG*) by nearly 1.2%. Additionally, we estimate that while a reduction in the GSR from the current 55% to 35% would decrease the overall producer participation rate from 95.5% to 84.4%, it would lower the expected government program cost share from 50.6% to 31.6%.

Due to their higher elasticities with respect to the GSR (table 2), most of the exiting producers would be those with the lowest yield risk (AFP of \$12/acre or less). Since the lower-risk producers are better equipped to stay in business in the face of adverse growing conditions, reducing the subsidy rates from 55% to 35% might not have a significant detrimental effect on long-run crop acreage and supply. While we recognize that these findings rely on some sparsely substantiated assumptions and pertain to the specific case of corn production in Iroquois County, Illinois, RMA decision makers might find this methodology useful to derive elasticity estimates based on their own information and assumptions.

Next, we illustrate how our model could be used by the RMA to assess the merits of alternative premium estimation methods for potentially improving program performance. We start by noting that the estimation protocol utilized by the RMA sets farm-level rates using an exponential “shrinkage” procedure that compresses the premium estimates implied by the individual farm yield data toward the county average (Milliman and Robertson, Inc., 2000). In the case of corn production in Iroquois County, this estimator renders a nearly identical premium quote with little variability across producers regardless of their reported APH. As a result of this shrinkage, that quote can be substantially biased if the AFP of the farm unit in question is much lower or higher than the county average.

A possible alternative to consider would be an estimator that is solely based on farm- or plot-level information such as long-term yield data, variety, soil and management system characteristics, etc. Some private crop insurance companies have developed and are using such models for their own risk ratings. This type of estimator could have a much lower bias than the RMA’s but would likely exhibit substantially higher variability around its expected value (Ramírez, Carpio, and Rejesus, 2011). Additionally, since the producer is also likely to base his or her own AFP estimate on the same information set, there should be at least a moderate correlation between the producer and the insurer premium estimates ($\rho_{12} = 0.50$). We thus assume that an estimator can be designed to have no bias (i.e., $\mu_2 = 0$) but a relatively high variance ($\sigma_2 = AFP/3$). As in the previous scenario, $RPP=1.10$, $\mu_1 = 0$, and $\sigma_1 = AFP/4$. However, we actually reduce the GSR from 0.55 to 0.325.

The outcomes corresponding to this second scenario are presented in the last six columns of table 2. Despite of the much lower GSR (32.5% versus 55%) and *PFG* (31.7% versus 50.6%), the use of this estimator is predicted to result in approximately the same overall *PPR* (95.3% versus 95.5%)

³ This participation rate should be an upper bound for what is observed in practice since producers have other coverage level choices (with different GSRs) that could be more appealing than 75%.

as the RMA's estimator. Also notably, all producers have the same *POP* (95.3%) regardless of their underlying *AFP*, which means that this estimator would not engender an adverse selection problem. Another salient characteristic is that the predicted distribution of the expected per acre subsidies is tighter under this estimator, ranging from \$2.331/acre (versus -\$0.789/acre) at an *AFP* of \$7/acre to \$10.656/acre (versus \$24.211/acre) at an *AFP* of \$32/acre. Furthermore, the percentage of the total indemnities paid by the government (*PFG*) is the same (33.3%) across all producer risk profiles (i.e., *AFPs*), versus a range of -11.3–75.7% for the RMA's (table 2). However, a drawback of using an estimator with these characteristics is that some farmers would inevitably receive very high premium quotes while others could pay next to nothing for crop insurance. Finally, under this estimator, the aggregate elasticity of the *POP* with respect to the *GSR* (0.150%) is smaller than under the RMA's (0.219%); therefore, a reduction in the *GSR* would have a less detrimental impact on participation.

Next, we assess how the performance of this alternative estimator would be affected if it exhibited a moderate systematic negative or positive bias. This is accomplished by setting $\mu_2 = -0.10 \times AFP$ (negative bias) and $\mu_2 = 0.10 \times AFP$ (positive bias). For the same *GSR* assumed in the no-bias scenario (32.5%), the negative bias increases the *PFG* from 31.7% to 38.7%, but it also improves the *POP* from 95.3% to 97.34%. Since the aggregate elasticity with respect to the *GSR* is only 0.0872%, one can substantially reduce the *GSR* without much impact on the *POP*. In fact, at a *GSR* of 26.5%, the *POP* scales back to 95.3% and the *PFG* stands at 33.2% (i.e., only 1.5% higher than under the no bias scenario).⁴ If the negative bias is raised to 25% (i.e., $\mu_2 = -0.25 \times AFP$), a *GSR* of just 15.5% is required to maintain a 95.3% participation rate, but the *PFG* increases to 36.3%. In other words, the total amount of subsidies that the government would have to provide is 14.5% higher due to this severe negative bias. Nevertheless, this larger 36.3% *PFG* is still much lower than the 50.6% predicted for the RMA estimator.

In contrast, when there is a moderate positive bias ($\mu_2 = 0.10 \times AFP$), the model predicts that 95.3% participation can be maintained with a *GSR* of 37.8% and a *PFG* of 30.7%. If this bias is raised to 25% ($\mu_2 = 0.25 \times AFP$), the required *GSR* increases to 44.5% but the *PFG* declines to 29.5%. In other words, it appears that a large positive bias in the insurer premium estimate could noticeably reduce the cost of maintaining a desired *PPR*. Empirically, this suggests that it might be desirable for the insurer to at least make sure that there is no negative bias on its premium estimates under a subsidized crop insurance scheme.

In the last two cases, we assumed that the bias was a fixed proportion of the underlying *AFP*. In this final scenario we explore the impact of a random bias by setting $\mu_2 = AFP(RU - 0.5)/5$, where *RU* is a random draw from a uniform distribution. If, for example, *AFP* = 10, then μ_2 would range from -1 to 1 with a uniformly equal probability. Under these conditions, a *GSR* of 32.7% is needed to sustain 95.3% participation and the resulting *PFG* is 31.9%, which is almost identical to the value obtained when no bias was assumed (i.e., $\mu_2 = 0$).

When a larger random bias is assumed ($\mu_2 = AFP(RU - 0.5)/2$) so that if, for example, *AFP* = 10, μ_2 uniformly ranges from -2.5 to 2.5 and a somewhat higher *GSR* (35%) is required to achieve the target 95.3% participation rate and the *PFG* increases from 31.7% in the no bias scenario to 34%. Thus, even if it is extreme in magnitude, the model predicts that this type of random bias would only have a moderate impact on the program's cost. We also observe that such bias has only a trivial effect on the expected per acre subsidy and its volatility as well as on the aggregate derivatives and elasticities with respect to μ_1 , σ_1 , μ_2 , σ_2 , ρ_{12} , the *GSR*, and the *RPP*.⁵

Concluding Remarks

With additional information or tailored assumptions about some of its key parameters, the theoretical model proposed in this study could be used by the RMA and other interested parties to evaluate how

⁴ Tables with the detailed statistics corresponding to these scenarios are available from the authors upon request.

⁵ Tables with the detailed statistics corresponding to these scenarios are available from the authors upon request.

the performance of the crop insurance program would be affected by changes in premium subsidy rates, the characteristics/properties of the estimation protocol, farmers' ability to more accurately assess their own yield risks, the magnitude of the risk protection premium they are willing to pay, etc. The proposed model could also be used by the RMA to refine its current protocols or assess the merits of alternative premium estimation methods for improving program performance. Obviously, the RMA has much more detailed and complete information and resources to improve on the accuracy of our assumptions and thus to obtain more reliable results.

Although our empirical application is for the specific case of corn in Iroquois County, Illinois, and we had to make some major assumptions about the mean and the variance of the producer premium estimates, our main results stem from the fact that the RMA protocol uses an exponential adjustment to substantially shrink premium estimates toward the county mean. This particular feature of RMA ratemaking creates a substantial bias in the estimates that happens to be strongly correlated with the underlying fair premium (i.e., if the AFP is low (high) the bias is positive (negative)) (table 1).

Because of this characteristic of the RMA premium estimation protocol, our model predicts that high-risk farmers could on average be receiving much larger subsidies than low-risk producers, and we expect high-risk farmers to be substantially more likely to purchase insurance than low-risk individuals, creating what is commonly referred to as the adverse selection problem. Theoretically, we show that alternative premium estimation methods could achieve similarly high levels of participation without an adverse selection problem and could provide a more equitable distribution of the subsidies across the participating producers at a much lower overall cost to taxpayers. Even if those estimators were to exhibit a significant amount of bias and large levels of premium estimation error, they would continue to perform well as long as that bias were not correlated with the underlying AFP.

Our model also suggests that under the current protocol, further subsidy increases might have much smaller impacts on program enrollment as participation expands (i.e., additional percentage gains in participation could become much more costly to taxpayers). On the other hand, if subsidy levels were to be reduced, our model predicts that most farmers exiting the program would be those with lower yield risk. Since they are much better equipped to stay in business in face of adverse growing conditions, even a substantial reduction on the subsidy rate might not have a significant detrimental effect on long-run crop acreage and supply.

Another important finding is that, in contrast to past literature arguing that high subsidies are needed in part because farmers have more accurate yield risk information than the RMA, increased producer knowledge about correct premiums should enhance (not hinder) the actuarial performance of the program. We also show that producer error has a much larger impact on participation than insurer error; thus, educational programs aimed at improving farmers' capacity to assess yield risks could increase enrollment in crop insurance without additional subsidies.

As well, everything else being constant, a higher correlation between the producer and the insurer premium estimate makes it more likely for a producer to purchase crop insurance. Thus, educational programs that enhance farmers' trust in the accuracy of the RMA premium quotes or the use of alternative premium estimation methods that are better correlated with what the producer thinks is a correct premium could sustain participation at a potentially lower cost to taxpayers. Further, a higher correlation would also reduce inequity in the distribution of subsidies across participating producers.

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