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U.S. Sweetener Demand Analysis: A QUAIDS Model Application

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We estimate the expenditure, price, and Engel parameters for the major U.S. caloric sweeteners (sugar, high-fructose corn syrup [HFCS], and glucose), for the 1975–2013 period using the quadratic almost ideal demand system (QUAIDS). The estimated parameters are then used to compute expenditure elasticities and both uncompensated and compensated price elasticities. We find that consumer expenditures are positively elastic for both sugar and HFCS but not for glucose. The own-price elasticity of demand for sugar is less elastic compared to those of HFCS and glucose. Our results will help design an effective U.S. sweetener tax policy.

Key words: endogeneity, glucose, HFCS, negative semi-definiteness, separability, Slutsky matrix, sugar

Introduction

The changing nature of sweetener consumption patterns in the United States presents challenges and opportunities as sweetener regulation and tax policy evolve. Sweeteners are broadly classified as caloric, low-caloric, or noncaloric (high-intensity) sweeteners. Caloric sweeteners include sugar, high-fructose corn syrup (HFCS), glucose, honey, and dextrose. Low-caloric sweeteners include sugar alcohols (sorbitol and mannitol), and noncaloric sweeteners include natural (stevia and monk fruit) and artificial (aspartame, sucralose, and neotame) sweeteners. Caloric sweeteners constitute the majority of sweeteners consumed in the United States. At the same time, noncaloric sweeteners (e.g., stevia) are gaining popularity among consumers.

HFCS was introduced in the 1970s as a cheap sweetener alternative as a response to high sugar prices. Because HFCS can be substituted for sugar in beverages, jams, and jellies but not in chocolates and confectionaries (Moss and Schmitz, 2002), we hypothesize that sugar and HFCS are imperfect substitutes. Between 1980 and 2000, HFCS replaced sugar as the main sweetener used in the beverage industry. Since 2000, however, HFCS consumption has decreased relative to sugar, partly due to HFCS's potential link to the obesity epidemic (Bray, Nielsen, and Popkin, 2004). Total per capita consumption of caloric sweeteners peaked at 89.3 pounds in 1999 (figure 1) before dropping to 75.4 pounds in 2013, due mainly to the decrease in HFCS consumption. Per capita HFCS consumption declined from 37.4 pounds in 1999 to 25.7 pounds in 2013, while sugar consumption increased slightly from 37.2 pounds per capita in 2002 to 39.9 pounds per capita in 2013 (figure 1).

This study empirically estimates the uncompensated and compensated price and expenditure elasticities for caloric sweetener (sugar, HFCS, and glucose) demand. Sugar, HFCS, and glucose

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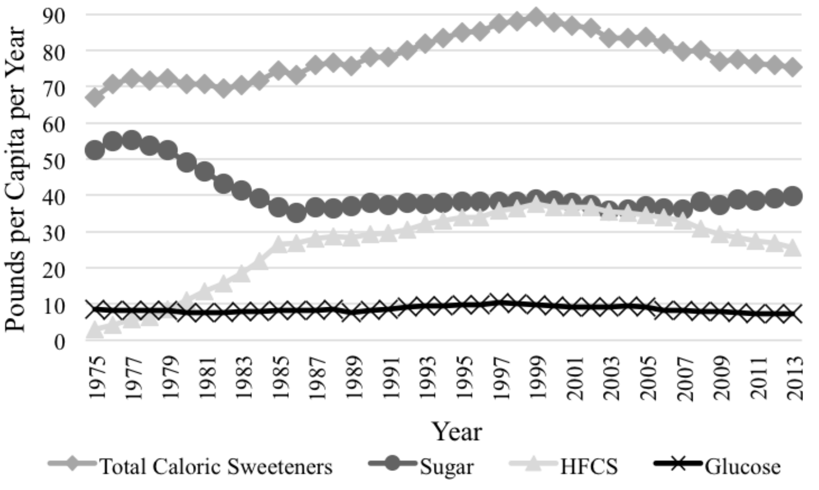

Figure 1. U.S. Caloric Sweetener Consumption, 1975–2013

Notes: Total caloric sweeteners data include dextrose, honey, and other caloric sweeteners.
Source: Data collected from USDA sugar and sweetener yearbook tables.

together constituted about 93% of total caloric sweetener consumption in the United States between 1975 and 2013 (authors' computations from USDA sweetener yearbook tables).

Many previous studies of sweetener demand have dealt with dynamic almost ideal demand system (AIDS) models or a linear approximation of the Quadratic AIDS (QUAIDS) model. For instance, to assess habit formation Zhen et al. (2011) used a dynamic AIDS model to estimate demand for sugar-sweetened beverages (SSBs) in low-income and high-income household groups. For low-income households, they estimated own-price (short-run) elasticities to be −1.22 and −1.91 for regular and diet soft drinks, respectively. Similarly, for high-income households, they estimated own-price (short-run) elasticities of −1.44 and −1.29 for regular and diet soft drinks, respectively. Finally, they found that the SSB tax would result in a moderate decrease in consumption for both types of households.

Dharmasena and Capps (2012) used a linear approximation in the QUAIDS model to capture the demand interrelationships among ten nonalcoholic beverages and to analyze the effect of a SSB tax on reduction in weight and calories consumed. They estimated own-price elasticities to be −2.25, and −1.27 for regular and diet soft drinks, respectively. They also found that a 20% tax on SSBs would result in a reduction of between 1.54 and 2.55 pounds of body weight per year. Miao, Beghin, and Jensen (2012) studied two tax policies (input tax and consumption tax) and their impact on added sweetener consumption. The authors recovered the parameters of their LinQuad incomplete demand system using income and price elasticities from the USDA/ERS commodity and food elasticities database (U.S. Department of Agriculture, Economic Research Service, 2008). They used −0.93 as the own-price elasticity for soft drinks and found that an input tax on added sweeteners is more efficient than a consumer tax because an input tax minimized the loss in consumer surplus.

Most previous studies (Zhen et al., 2011; Miao, Beghin, and Jensen, 2012; Dharmasena and Capps, 2012; Zhen et al., 2014) analyzed the demand for SSBs, but none estimated a comprehensive sweetener demand model. This study further contributes to the literature on sweetener demand by estimating the sweetener demand system, including major caloric sweeteners, through the QUAIDS model.

Sweetener taxation is growing in popularity in the United States and elsewhere. In the Berkeley, California, a soda tax has been in effect since January 1, 2014. Other soda tax policies are still being debated, including the "SWEET Act" (U.S. Congress, 2015). More recently, soda tax proposals have been considered by other U.S. cities. In addition, countries such as Mexico and the United Kingdom have been successful in implementing a soda tax. However, many argue that taxing just

SSBs, which account for only about 35.7% of total U.S. sweetener consumption, is insufficient to attain the desired goal (U.S. Department of Agriculture and U.S. Department of Health and Human Services, 2010). A comprehensive sweetener tax, irrespective of the product or the sweetener used, would be an ideal solution. It is in this context that we use the term "sweetener tax" instead of "soda tax."[1]

This study contributes to the literature in two important ways. First, this study quantifies the demand structure of sweeteners; the derived elasticity results could be used for policy analysis. Second, we derive weak separability restrictions for the QUAIDS model and perform the test for appropriate commodity aggregation in the demand system. Testing for weak separability is important as it helps in understanding the nature of commodity aggregation for performing demand analysis.

The results of this study have significant policy implications, especially because sweetener taxation is growing in popularity. Own-price and cross-price elasticities play a significant role in evaluating the impact of price changes on the quantity of sweeteners under consideration when designing policies related to sweetener use. The reliability of the elasticities is important for policy analyses of sweeteners. Demand estimation models that do not control for price and expenditure endogeneity may lead to biased parameter estimates and unreliable demand elasticities. The QUAIDS model used in this study incorporates quadratic Engel curves for expenditures and accounts for price and expenditure endogeneity. The model also imposes local curvature restrictions to ensure negative semi-definiteness of the Slutsky matrix.

Model Specification

Quadratic Almost Ideal Demand System (QUAIDS) and Test Specification

We use the quadratic almost ideal demand system (QUAIDS) originally proposed by Banks, Blundell, and Lewbel (1997) to estimate the elasticities of major sweeteners. The uncompensated (and compensated) own-price elasticities, cross-price elasticities, and expenditure elasticities are computed using QUAIDS. The QUAIDS model specification has an indirect utility function (V) of the form

$$\ln V = \left\{ \left[\frac{\ln m - \ln(P)}{b(p)} \right]^{-1} + \lambda(p) \right\}^{-1}. \tag{1}$$

The QUAIDS specification builds on the traditional almost ideal demand system (AIDS) developed by Deaton and Muellbauer (1980). In the traditional model proposed by Deaton and Muellbauer (1980), the indirect utility function ($\ln V = \left[\frac{\ln m - \ln(P)}{b(p)}\right]$) was derived from price independent, generalized logarithmic preferences (PIGLOG). Later, Banks, Blundell, and Lewbel (1997) proposed equation (1) with $\lambda(p)$ added to the original indirect utility equation to account for the effect of quadratic Engel curves on the budget/expenditure shares.[2] In general, the Engel curve describes how the expenditure share of a good varies with income. Banks, Blundell, and Lewbel (1997) found that Engel curves are linear for necessities (such as food) and nonlinear for other commodities such as alcohol and clothing.

We consider three goods—sugar, HFCS, and glucose—in the model. The expenditure/budget share of good i is of the form (Banks, Blundell, and Lewbel, 1997)[3]

$$w_i = \alpha_i + \sum_{j=1}^{n} \gamma_{ij} \ln p_j + \beta_i \ln\left[\frac{m}{P}\right] + \frac{\lambda_i}{b(p)} \left\{ \ln\left[\frac{m}{P}\right] \right\}^2, \tag{2}$$

[1] A soda or SSB tax is levied only on a specific product (e.g., soda or SSBs), while a sweetener tax is levied on any product irrespective of the sweetener used or present in that product. Most SSBs in the United States use HFCS as the main sweetener.

[2] $\lambda(p)$ is a function differentiable and homogeneous in prices.

[3] For detailed derivations of expenditure/budget share equations refer to Banks, Blundell, and Lewbel (1997).

where α_i, β_i, γ_{ij}, and λ_i are parameters; m is total expenditure; p_j is the price of good j; and $\ln(P)$ and $b(p)$ are translog and Cobb–Douglas price aggregator functions, respectively. The translog price aggregator is written as $\ln(P) = \alpha_0 + \sum_{i=1}^{n} \alpha_i \ln p_i + \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n} \gamma_{ij} \ln p_i \ln p_j$, and the Cobb–Douglas price aggregator is written as $b(p) = \Pi_{i=1}^{n} p_i^{\beta_i} = \exp(\sum_i \beta_i \ln p_i)$. In empirical applications, it is difficult to compute α_0 of the translog price aggregator function, $\ln(P)$. Assigning α_0 at a certain value has been well documented in the literature. For example, Banks, Blundell, and Lewbel (1997, p. 534) claim that α_0 is chosen to be just below the minimum of log expenditure given its interpretation as the cost of a minimum standard of living (Deaton and Muellbauer, 1980). In our case, α_0 could be interpreted as the cost of minimum sweeteners consumed in the United States. Therefore, we set the value of α_0 at $0.95 \times \min\{\ln(m)\}$ as suggested by Hovhannisyan and Gould (2011).[4]

Theoretical restrictions of adding up ($\sum_i \alpha_i = 1$, $\sum_i \beta_i = 0$), homogeneity ($\sum_j \gamma_{ij} = 0$, $\sum_i \lambda_i = 0$), and Slutsky symmetry ($\gamma_{ij} = \gamma_{ji}$) (for all j not equal to i) are imposed on the demand system.

The prices and expenditures used in the study may be endogenous in sweetener demand equations. Hence, we follow Dhar, Chavas, and Gould (2003) to test for price and expenditure endogeneity using a Durbin–Wu–Hausmann test. To prevent curvature violations, we also impose local curvature conditions in the system using the method proposed by Ryan and Wales (1998).

Price and Expenditure Endogeneity

We follow Dhar, Chavas, and Gould (2003) to account for price and expenditure endogeneity in the demand system. We specify reduced form equations for each of the commodity prices. For sugar (p_s), we use the sugar producer price index (*sppi*) as the exogenous variable because it affects the supply of sugar. For all other corn sweeteners—HFCS (p_h) and glucose (p_g)—we use the corn price (*cornp*) as the exogenous variable.[5] We specify the logarithm of sweetener prices as dependent variables. Upon checking the validity of both the instruments—sugar producer price index and corn price—we found that they are (1) correlated with their respective dependent variables (logarithms of sugar price, HFCS price, and glucose price) and (2) uncorrelated with the residuals of their respective sweetener expenditure share equations. After satisfying the above two conditions, we are confident that logarithms of sugar producer price index and corn price are exogenous variables. We specify the logarithms of prices as dependent variables and logarithms of exogenous variables as independent variables in the reduced-form equations:[6]

$$\ln p_s = f(constant, \ln sppi); \tag{3}$$

$$\ln p_h = f(constant, \ln cornp); \tag{4}$$

$$\ln p_g = f(constant, \ln cornp). \tag{5}$$

The logarithm of per capita income (*pcinc*) and the translog price index ($\ln(P)$) were used as exogenous variables for the reduced-form expenditure:[7]

$$\ln m = f(constant, \ln pcinc, \ln(P)). \tag{6}$$

[4] One of the reviewers pointed out that the results can be sensitive to the chosen value of α_0. Hence, we carried out a sensitivity assessment of α_0 with consecutive values between 0.93 and 1.00, each value multiplied with $\min\{\log(m)\}$ at a time. The results did not change significantly.

[5] Corn is the raw material used in the production of HFCS and glucose; hence, corn price is an important determinant of the price of these sweeteners.

[6] We used linear trend as one of the independent variables in the reduced form equations (to capture the time specific unobservable effects) but found to be insignificant.

[7] Alston, Chalfant, and Piggott (2001) recommended using a generalized version of QUAIDS when the expenditure share equations include demographic or seasonal variables in order to maintain the invariance of the unit of measurement. Given the aggregate (U.S.-level yearly data) nature of our analysis, we do not use seasonal or demographic variables in the expenditure share equations and hence rely on QUAIDS instead of generalized QUAIDS.

Table 1. Structure of Separable Demand Models

Commodity	**Separable Groupings** [1]	[2]
Sugar	A	A
HFCS	B	A
Glucose	B	B
No. of Commodity Groups	2	2
No. of Joint Tests/Non-Redundant Restrictions	1	1

Notes: For computing the number of non-redundant restrictions refer to Moschini, Moro, and Green (1994) and (Sellen and Goddard, 1997).

We adopt a full information maximum likelihood (FIML) estimation procedure to account for both price and expenditure endogeneity. This approach augments the expenditure share equations with four reduced-form equations of price and expenditure and jointly estimates them in order to obtain the parameter estimates (Hovhannisyan and Gould, 2011).

We follow the Durbin–Wu–Hausmann (DWH) test to test for price and expenditure endogeneity in the demand system. The null hypothesis is that the parameter estimates are consistent without controlling for endogeneity (Dhar, Chavas, and Gould, 2003). The test statistic H is computed as

$$H = (\beta_{ISUR} - \beta_{FIML})[VAR(\beta_{ISUR}) - VAR(\beta_{FIML})]^{-1}(\beta_{ISUR} - \beta_{FIML}), \tag{7}$$

where β_{ISUR} is the vector of estimated coefficients without controlling for endogeneity by iterative, seemingly unrelated regression, and β_{FIML} is the vector of estimated coefficients after controlling for price and expenditure endogeneity by a full information maximum likelihood estimation. In the test procedure, H is asymptotically distributed as a chi-square statistic, $\chi(k)$, where k is the degrees of freedom equal to the number of parameters being tested. The value of k can be obtained from the $[VAR(\beta_{ISUR}) - VAR(\beta_{FIML})]$ matrix, where k is the number of positive diagonal elements of the differenced variance-covariance matrix (Hall and Cummins, 2009, pp. 192–193). In this study, for testing price and expenditure endogeneity, $k = 9$.

Testing for Weak Separability

We also test for weak separability among different sweeteners by using the test proposed by Moschini, Moro, and Green (1994). We derive the weak separability restrictions for the QUAIDS model.

We test for nonhomothetic weak separability using the approach developed by Moschini, Moro, and Green (1994). As Sellen and Goddard (1997) state, weak separability implies that "the marginal rate of substitution between two consumption goods in one group is independent of quantities of goods consumed from outside the group" (p. 133). Two separable structures are selected for testing weak separability, as shown in table 1. These separable groups are formed based on (1) the source of sweetener and (2) major sweetener. In separable structure [1], sugar is formed as one group (group A) and both HFCS and glucose, which are derived from corn, are formed as another group (group B). Additionally, in separable structure [2], sugar and HFCS are placed in group A and glucose is placed in group B.

According to Moschini, Moro, and Green (1994), the following restrictions apply to the elasticities:

$$\frac{\sigma_{ik}}{\sigma_{jm}} = \frac{\epsilon_i \epsilon_k}{\epsilon_j \epsilon_m}, \tag{8}$$

where σ_{ik}, σ_{jm}, ϵ_i, ϵ_k, ϵ_j, and ϵ_m are the elasticity of substitution between good i and good k; the elasticity of substitution between good j and good m; and the expenditure elasticities of good i, good k, good j, and good m, respectively. Since we have only one good in one of the groups of each structure, the equation (8) simplifies to

$$\left[\frac{\sigma_{ik}}{\sigma_{jk}} = \frac{\epsilon_i}{\epsilon_j}\right]. \tag{9}$$

At the sample means, the parametric restrictions are

$$\begin{aligned}&\frac{\gamma_{ik}-\alpha_0(\beta_i\beta_k+\alpha_i\beta_k+\alpha_k\beta_i)+\alpha_0^2(\beta_i\lambda_k+\lambda_i\beta_k+\lambda_k\alpha_i+\beta_i\beta_k+\lambda_i\alpha_k)-\alpha_0^3(2\lambda_i\lambda_k+\lambda_k\beta_i+\lambda_i\beta_k)+\alpha_0^4\lambda_i\lambda_k+\alpha_i\alpha_k}{\gamma_{jk}-\alpha_0(\beta_j\beta_k+\alpha_j\beta_k+\alpha_k\beta_j)+\alpha_0^2(\beta_j\lambda_k+\lambda_j\beta_k+\lambda_k\alpha_j+\beta_j\beta_k+\lambda_j\alpha_k)-\alpha_0^3(2\lambda_j\lambda_k+\lambda_k\beta_j+\lambda_j\beta_k)+\alpha_0^4\lambda_j\lambda_k+\alpha_j\alpha_k} = \\ &\frac{\beta_i+\alpha_i+\alpha_0(-2\lambda_i-\beta_i+\lambda_i\alpha_0)}{\beta_j+\alpha_j+\alpha_0(-2\lambda_j-\beta_j+\lambda_j\alpha_0)}.\end{aligned} \tag{10}$$

Finally, these parametric restrictions are imposed on the quadratic demand system (equation 2) and tested against the unrestricted model using a likelihood ratio test. We also test the unrestricted with the restricted model using the size-corrected likelihood ratio.

Moschini, Moro, and Green (1994) applied weak separability restrictions for a full nonlinear AIDS model. In this study, our restrictions are different from their study in two ways: (1) we did not assume that $\alpha_0 = 0$; instead we substituted the logarithm of minimum expenditure, as mentioned above, and (2) we derived our restrictions for the Quadratic AIDS model as opposed to the full nonlinear AIDS model (see the appendix for derivations of the weak separability restrictions imposed on the quadratic AIDS model).

Imposing Local Curvature Restrictions

In order to maintain negative semi-definiteness of the Slutsky matrix,[8] we impose local curvature restrictions in QUAIDS following the procedure proposed by Ryan and Wales (1998) and Chang and Serletis (2012). The first step in imposing local curvature restriction is to compute the elements of the Slutsky matrix. For the traditional AIDS model, the Slutsky matrix elements are computed with the following equation (Slottje, 2009):

$$s_{ij} = \gamma_{ij} + \beta_i\beta_j \ln\left(\frac{m}{P}\right) - w_i\delta_{ij} + w_i w_j. \tag{11}$$

Similarly, for computing the elements of QUAIDS Slutsky matrix, we use

$$\begin{aligned} s_{ij} = {} & \gamma_{ij} + \beta_i\beta_j \ln\left(\frac{m}{P}\right) + \frac{\lambda_i\beta_j}{b(p)}\left[\ln\left(\frac{m}{P}\right)\right]^2 + \frac{\lambda_j\beta_i}{b(p)}\left[\ln\left(\frac{m}{P}\right)\right]^2 \\ & + \frac{2\lambda_i\lambda_j}{[b(p)]^2}\left[\ln\left(\frac{m}{P}\right)\right]^3 - w_i\delta_{ij} + w_i w_j, \end{aligned} \tag{12}$$

where $\delta_{ij} = 1$ if $i = j$ and 0 otherwise.

Following the notation of Chang and Serletis (2012), we replace the Slutsky matrix (S_{ij}) with ($-KK'$), where K is the lower triangular matrix. We then estimate the FIML estimation model using the elements obtained after multiplying with the ($-KK'$) matrix with the restrictions at a reference point, say at $p = m = 1$. At this reference point, the budget/expenditure shares (w) are derived from equation (2):

(13) $w_i = \alpha_i - \beta_i\alpha_0 + \lambda_i\alpha_0^2$ and $w_j = \alpha_j - \beta_j\alpha_0 + \lambda_j\alpha_0^2$ for goods i and j, respectively.

[8] We initially performed our analysis without imposing the curvature restrictions and found that there are curvature violations.

Evaluating equation (12) by substituting w_i and w_j and solving for γ_{ij} at the reference point ($p = m = 1$), we get the following restrictions:

$$\begin{aligned}\gamma_{ij} &= (-KK')_{ij} + \beta_i\beta_j\alpha_0 - \lambda_i\beta_j\alpha_0^2 - \lambda_j\beta_i\alpha_0^2 + 2\lambda_i\lambda_j\alpha_0^3 + (\alpha_i - \beta_i\alpha_0 + \lambda_i\alpha_0^2)\delta_{ij} \\ &\quad - (\alpha_i - \beta_i\alpha_0 + \lambda_i\alpha_0^2) \times (\alpha_j - \beta_j\alpha_0 + \lambda_j\alpha_0^2).\end{aligned} \tag{14}$$

For instance, the restrictions for γ_{11}, γ_{12}, and γ_{22} are as follows (Chang and Serletis, 2012):[9]

$$\begin{aligned}\gamma_{11} &= -k_{11}^2 + \beta_1\beta_1\alpha_0 - 2\lambda_1\beta_1\alpha_0^2 + 2\lambda_1\lambda_1\alpha_0^3 \\ &\quad + (\alpha_1 - \beta_1\alpha_0 + \lambda_1\alpha_0^2) - (\alpha_1 - \beta_1\alpha_0 + \lambda_1\alpha_0^2)^2,\end{aligned} \tag{15}$$

$$\begin{aligned}\gamma_{12} &= -k_{11}k_{12} + \beta_1\beta_2\alpha_0 - \lambda_1\beta_2\alpha_0^2 - \lambda_2\beta_1\alpha_0^2 + 2\lambda_1\lambda_2\alpha_0^3 \\ &\quad - (\alpha_1 - \beta_1\alpha_0 + \lambda_1\alpha_0^2) \times (\alpha_2 - \beta_2\alpha_0 + \lambda_2\alpha_0^2),\end{aligned} \tag{16}$$

$$\begin{aligned}\gamma_{22} &= -k_{12}^{2-} - k_{22}^2 + \beta_2\beta_2\alpha_0 - 2\lambda_2\beta_2\alpha_0^2 + 2\lambda_2\lambda_2\alpha_0^3 \\ &\quad + (\alpha_2 - \beta_2\alpha_0 + \lambda_2\alpha_0^2) - (\alpha_2 - \beta_2\alpha_0 + \lambda_2\alpha_0^2)^2,\end{aligned} \tag{17}$$

where k elements are obtained after multiplying $(-KK')$ matrix as mentioned in Chang and Serletis (2012, pp. 42–43).

After accounting for price and expenditure endogeneity and imposing local curvature restrictions, we estimate the QUAIDS model with the FIML estimation to obtain the parameters of interest. These parameters are used to compute the required elasticities. For the QUAIDS model, we obtain expenditure elasticities (ϵ_i), uncompensated own-price elasticities, and uncompensated cross-price elasticities (ε_{ij}) with equations (18) and (19) (Banks, Blundell, and Lewbel, 1997):

$$\epsilon_i = \frac{\beta_i + \frac{2\lambda_i}{b(p)}\left\{\ln\left[\frac{m}{P}\right]\right\}}{w_i} + 1 \tag{18}$$

$$\varepsilon_{ij} = \frac{\gamma_{ij} - \left(\beta_i + \frac{2\lambda_i}{b(p)}\left\{\ln\left[\frac{m}{P}\right]\right\}\right)(\alpha_j + \sum_k \gamma_{jk}\ln p_k) - \frac{\lambda_i\beta_j}{b(p)}\left\{\ln\left[\frac{m}{P}\right]\right\}^2}{w_i} - \delta_{ij} \tag{19}$$

where δ_{ij} is the Kronecker delta, which equals 1 when $i = j$ and 0 otherwise.

Compensated own-price elasticities and cross-price elasticities are computed using

$$\mu_{ij} = \varepsilon_{ij} + (\epsilon_i \times w_j), \tag{20}$$

where ε_{ij} is the uncompensated elasticity from equation (19) and ϵ_i and w_j are the expenditure elasticity and expenditure share of good i, respectively.

Data

We collected annual U.S. price and quantity data from 1975 through 2013 for the per capita consumption of sugar, HFCS, and glucose from the USDA sugar and sweetener yearbook tables. Glucose syrup was converted from bulk quantities to per capita quantities by dividing it by the U.S. population for the time period considered. Population data were collected from the World Bank database (World Bank, 2014). A summary of descriptive statistics of sweetener prices, quantities, and expenditure shares are presented in table 2.

[9] Refer to Chang and Serletis (2012) for additional details on imposing local curvature restrictions.

Table 2. Descriptive Statistics of Sweeteners, 1975–2013

	Mean	Standard Deviation	Minimum	Maximum
Price (Cents/pound/year)				
Sugar	42.45	10.97	21.62	69.41
HFCS	17.58	4.73	10.58	28.70
Glucose	15.82	6.89	10.10	35.73
Quantity (Pounds per capita/year)				
Sugar	40.44	5.83	35.27	55.33
HFCS	26.37	10.28	2.87	37.49
Glucose	8.55	0.85	7.21	10.39
Expenditure Shares				
Sugar	0.75	0.07	0.66	0.93
HFCS	0.19	0.07	0.03	0.28
Glucose	0.06	0.01	0.03	0.07
Other Variables				
Corn Price ($/bu)	2.82	1.19	1.56	6.67
Sugar PPI	113.92	27.47	51.40	193.90
Income ($ in 1,000s)	44.12	18.70	13.78	72.64

Notes: For sugar and HFCS prices, we used refined sugar and HFCS-42 prices, respectively.

The U.S. mean per capita income data were collected from the U.S. Census Bureau database (2013). Data on the producer price index of sugar and corn prices were collected from the USDA database. The U.S. Department of Agriculture (2014) computed the actual quantity of sweeteners at the consumer level after accounting for losses at the wholesale, retail, and consumer levels. For example, the USDA reports an 11% loss of sweeteners from the retail level to the consumer. At the consumer level, the USDA reports a 34% loss of sweeteners due to spoilage, uneaten food, etc. We used quantity data (pounds per capita per year) at the consumer level after accounting for losses and performed the demand analysis of sweeteners. It is important to note that sweetener deliveries (to firms) data are different from sweetener consumption at the consumer level. Specifically, we used the consumer-level data from the USDA sugar and sweetener yearbook tables. Sweetener market prices (cents per pound per year) were collected from the USDA. Due to insufficient data, we did not include other caloric sweeteners such as dextrose and honey, which together account for about 2% of total caloric sweetener consumption in the United States.

Empirical Results and Discussion

The QUAIDS model is estimated using the FIML procedure. We use the TSP International software system to obtain various parameter estimates of the QUAIDS model. The QUAIDS model in equation (2) and the reduced form of equations (3)–(6) are jointly estimated to account for price and expenditure endogeneity. With the QUAIDS model, we estimate twenty-four parameters (in total) using six equations. The six equations include two expenditure share equations (parameters in the third expenditure share equation of glucose were computed using the adding-up restriction), three reduced-form price equations, and one reduced-form expenditure equation. Table 4 presents the parameter estimates of the QUAIDS model after accounting for price and expenditure endogeneity and after imposing the local curvature restrictions. The restricted parameters are computed through the ANALYZ command in the TSP. Of twenty-four parameters, twenty-one are significant at $p < 0.01$, one is significant at $p < 0.05$, and two are not significant.

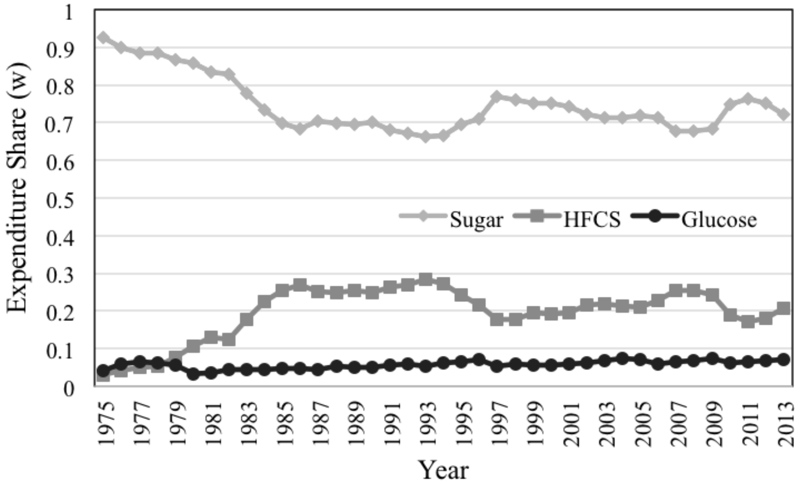

Figure 2. Expenditure Shares of Sweeteners, 1975–2013

Source: Data collected from USDA sugar and sweetener yearbook tables.

Table 3. Results of Weak Separability Tests

Separable Groupings	Number of Restrictions	LR Test Statistic	Size-Corrected LR Test Statistic	Critical Value ($\chi_{0.5}$)
[1]	1	142.10	120.22	3.84
[2]	1	149.74	126.68	3.84

Notes: Non-homothetic separability restrictions were imposed on the demand system.

For testing endogeneity, the benchmark model under the null hypothesis (no endogeneity) is the QUAIDS model after imposing symmetry and homogeneity constraints with nine parameters (the third expenditure share equation is dropped). This benchmark model is tested against a model allowing price and expenditure endogeneity augmenting the benchmark model with equations (3)–(6). The DWH endogeneity test is performed as shown in equation (7). The estimated DWH chi-square test statistic from equation (7) is found to be 208.48 ($p < 0.01$). Hence, controlling for price and expenditure endogeneity is required to consistently estimate the QUAIDS parameters. Therefore, any empirical work without accounting for endogeneity may yield biased and inconsistent parameter estimates and thereby produce unreliable elasticities.

Test results of weak separability are shown in table 3. The unrestricted model imposes homogeneity and symmetry and assumes no separability of the utility function. This unrestricted model is tested against the model with the separable structural groups mentioned earlier in table 1. We performed both a likelihood ratio test (LR) and a size-corrected LR test based on the suggestion of Moschini, Moro, and Green (1994). Results from the size-corrected LR test (and the LR test) show that the restricted model is significantly different from the unrestricted model at $p < 0.05$. Results of weak separability establish an appropriate commodity aggregation for use in sweetener demand systems, and it is implied that all commodities should be included simultaneously in the demand system.

All parameter estimates (table 4) of the reduced form of the price and expenditure equations are significant at least at the 1% level except for the constant parameter of sugar price equation. In the reduced form of expenditure equations, the per capita income parameter is significantly different from zero at $p < 0.05$. As predicted, all of the coefficient estimates of the reduced-form equations bear the expected signs. For instance, the input/supply price instruments—such as sugar producer price index and corn price parameters—are positively related to the price of sugar and corn sweeteners, respectively. Additionally, the income parameter is positively related to sweetener

Table 4. Parameter Estimates from the QUAIDS Model after Accounting for Price and Expenditure Endogeneity, 1975–2013

Parameters (1)	Sugar (2)	HFCS (3)	Glucose (4)
α	0.609***	0.807***	−0.417***
	(0.048)	(0.034)	(0.037)
β	−0.187***	0.173***	0.014
	(0.009)	(0.015)	(0.009)
γ Sugar	−0.078***	0.131***	−0.053***
	(0.025)	(0.025)	(0.009)
γ HFCS		−0.250***	0.119***
		(0.029)	(0.009)
γ Glucose			−0.066***
			(0.005)
λ	−0.055***	0.006***	0.049***
	(0.002)	(0.002)	(0.002)
Parameter Estimates from the Reduced-Form Equations of Price and Expenditures			
Constant–Sugar Price			−0.363
			(0.237)
Sugar–Producer Price Index			0.866***
			(0.050)
Constant–HFCS Price			2.406***
			(0.096)
Corn Price–HFCS			0.434***
			(0.084)
Constant–Glucose Price			2.062***
			(0.111)
Corn Price–Glucose			0.641***
			(0.097)
Constant–Expenditure			−5.214***
			(0.144)
Income			0.027**
			(0.011)
Translog Price Index			1.234***
			(0.019)

Notes: Double and triple asterisks (**, ***) indicate significance at the 5% and 1% level. Values in parentheses are standard errors.

expenditures. This result indicates that absolute sweetener expenditure increases (although share might decrease) with an increase in per capita income.

As shown in table 4, the Engel curves (λ) are quadratic for all commodities, implying that the QUAIDS model is more appropriate than the traditional AIDS model, which does not include the quadratic term in its specification. The Engel curve for sugar is negative and positive for HFCS and glucose. This result is not surprising, because the expenditure share of sugar has been decreasing over time while the expenditure share of HFCS has been increasing, as shown in figure 2. In addition, this result is evident from the β_i coefficients in table 4, where the expenditure parameter, β_i, is negative for sugar and positive for HFCS.

Own-price (γ_{ii}) and cross-price (γ_{ij}) parameters are reported along the diagonal and nondiagonal elements (columns (2)–(4) and rows γ in table 4), respectively. All the own-price and cross-price parameters of sugar, HFCS, and glucose are significant at $p < 0.01$ (table 4). We used the delta method to obtain the standard errors of the elasticity estimates.

Table 5. Uncompensated Elasticity Matrix of U.S. Sweeteners after Accounting for Price and Expenditure Endogeneity, 1975–2013

	Sugar	HFCS	Glucose	Expenditure
Sugar	-1.32^{***}	0.10^{***}	0.02	1.19^{***}
	(0.042)	(0.024)	(0.024)	(0.019)
HFCS	0.34^{***}	-2.95^{***}	0.92^{***}	1.69^{***}
	(0.123)	(0.125)	(0.046)	(0.030)
Glucose	2.95^{***}	5.34^{***}	-4.44^{***}	-3.85^{***}
	(0.452)	(0.191)	(0.251)	(0.202)

Notes: Triple asterisks (***) indicate significance at the 1% level. Values in parentheses are standard errors. Elasticities are evaluated at mean of the data.

Table 6. Compensated Elasticity Matrix after Accounting for Price and Expenditure Endogeneity, 1975–2013

	Sugar	HFCS	Glucose
Sugar	-0.42^{***}	0.33^{***}	0.09^{***}
	(0.030)	(0.023)	(0.024)
HFCS	1.60^{***}	-2.62^{***}	1.01^{***}
	(0.112)	(0.126)	(0.047)
Glucose	0.07	4.59^{***}	-4.66^{***}
	(0.308)	(0.195)	(0.262)

Notes: Triple asterisks (***) indicate significance at the 1% level. Values in parentheses are standard errors. Elasticities are evaluated at mean of the data.

Table 5 shows the expenditure elasticity matrix at the means of the data. These expenditure elasticities for a good represent the percentage change in quantity of that good demanded when total U.S. expenditures on sweeteners increase by 1%. Based on the expenditure elasticity results, both sugar and HFCS consumption increases by more than 1% when expenditure on sweeteners increase by 1%. It is important to note that the QUAIDS expenditure elasticity of a good depends on both the absolute value and the signs of β, λ, and w (from equation 18). In case of sugar, the consumer expenditure is elastic (1.19) due to the large, negative value of the $\left\{\ln\left[\frac{m}{P}\right]\right\}$ term (in the expenditure elasticity formula), despite both β and λ being negative. On the other hand, despite the fact that both the expenditure parameter and the Engel parameter of HFCS are positive, the consumer expenditure is elastic (1.69) because of a very low absolute value of λ, which gives the whole numerator a positive value (> 1) in the expenditure elasticity formula. In the last decade, regardless of the decreased consumption, HFCS maintained its expenditure share due to high prices. The high expenditure elasticity (1.69) of HFCS (compared to sugar) may also be due to its very high elasticity in the initial decade of the study period, where its expenditure share was as low as 0.03.

Eleven of the twelve elasticity estimates are significant at the 1% level (table 5). The negative sign of the own-price elasticities of all sweeteners is consistent with standard economic theory. The uncompensated own-price elasticities measure both the substitution effects and income effects of a change in price, whereas the compensated elasticities measure only the substitution effect of a change in price (Seale, Marchant, and Basso, 2003). Own-price elasticity results shown in both tables 5 and 6 suggest that glucose is highly elastic compared to HFCS and sugar. Sugar is the least own-price elastic of all the sweeteners considered in the study.

The cross-price elasticities measure the change in quantity of consumption of one good when the price of the other good is increased by one unit. The positive cross-price elasticities indicate substitution between the sweeteners. The cross-price elasticities of sugar–HFCS (0.10) and glucose–HFCS (5.34) indicate that if the HFCS price is increased by one unit, substitution occurs more with glucose (or maybe with other corn sweeteners) compared to sugar, which supports the claim that sugar and HFCS are imperfect substitutes.[10] This result—sugar and HFCS as imperfect substitutes—is significant in the context of sugar associations' lawsuit in the federal court of California against the corn refiners association for advertising HFCS as a natural product similar to sugar (Lipton, 2014; Globe Newswire, 2015).

The results of this study can be used for designing an effective sweetener tax policy. For example, the American Heart Association (AHA) recommends the daily per capita sweetener consumption of no more than six teaspoons (for women) to nine teaspoons (for men). Current daily U.S. per capita sweetener consumption is approximately nineteen teaspoons (authors' computations from USDA data). As excess sweetener consumption has been linked to health-related issues such as obesity, Type 2 diabetes, and cardiovascular disease (Johnson et al., 2009), there is a growing debate (and popularity) around taxing sweeteners, in the form of a soda tax, to reduce consumption in the United States and Mexico. A few examples include the 2014 soda taxes enacted in Berkeley, California, and Mexico. Many argue that taxing only sugar-sweetened beverages would be insufficient since they only account for 35.7% of total sweetener use in the United States (U.S. Department of Agriculture and U.S. Department of Health and Human Services, 2010). Other measures under debate include cap-and-trade on added sugars (Lewis and Basu, 2015) and the Food and Drug Administration's proposal to limit sugar to 10% of the Daily Value (DV) listed on nutrition labels (Watson, 2015). Hence, as sweetener tax policies evolve, it would be important to understand the effect of an increase in the price of sweeteners on consumers and their response.

There are certain caveats of our study. First, we could not test for weak separability of sweeteners from other goods due to data limitations and thereby could not report unconditional elasticity estimates. However, following Carpentier and Guyomard (2001), there is little deviation from conditional and unconditional elasticity estimates. There is a 2–9% deviation between conditional and unconditional own-price elasticity estimates for four of the five commodities (Carpentier and Guyomard, 2001, p. 228).[11] It is important to note that the difference between conditional and unconditional elasticities could be case-specific. Second, our study focused only on major caloric sweeteners (sugar, HFCS, and glucose) and did not account for other caloric (e.g., honey), low-caloric, and noncaloric sweeteners in the United States due to insufficient data. However, sugar, HFCS, and glucose account for about 93% of total caloric sweetener consumption in the U.S. (low-caloric and noncaloric sweeteners together account for about less than 2% of total sweetener consumption in the United States).

Future work might consider the impact of a tax on major sweeteners using their own-price elasticities in order to reduce their consumption. For instance, the own-price elasticities of sugar and HFCS could play a significant role in demand schedules (of policy analyses) for levying taxes on sweeteners to reduce their consumption level to the recommended level. Other future work might consider sweeteners that are separable from all other goods consumed based on the availability of the data.

Conclusions

We conducted a QUAIDS analysis of the major caloric sweeteners in the United States: sugar, HFCS, and glucose. An FIML estimation was used for parameters estimated after accounting for price, expenditure endogeneity, and weak separability for commodity aggregation. Our key findings are that (1) both sugar and HFCS are positively expenditure elastic, while glucose is negatively expenditure elastic (this indicates that only sugar and HFCS gain with an increase in sweetener expenditures); (2) the own-price elasticity results suggest that HFCS and glucose are highly elastic compared to sugar; and (3) the cross-price elasticity results show that substitution

[10] One of the reviewers stressed that high cross-price elasticities of sugar-glucose and HFCS-glucose may be partly due to imposing symmetry restrictions and a small expenditure share for glucose.

[11] See Carpentier and Guyomard (2001, p. 228) for their elasticity estimates of milk products.

favors HFCS-glucose compared to HFCS-sugar, supporting the claim that sugar and HFCS are imperfect substitutes.

The implications of these results are far reaching. While certainly controversial, taxing caloric sweeteners directly has the potential to significantly reduce U.S. consumption of sweeteners. Considering that current U.S. per capita consumption of caloric sweeteners far exceeds the daily recommendation of the American Heart Association and that prolonged excess consumption of sweeteners has adverse effects on health, our results should help policy makers decide on the tax levels needed to achieve a reduction in sweetener consumption.

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References

Alston, J. M., J. A. Chalfant, and N. E. Piggott. "Incorporating Demand Shifters in the Almost Ideal Demand System." *Economics Letters* 70(2001):73–78. doi: 10.1016/S0165-1765(00)00322-0.

Banks, J., R. Blundell, and A. Lewbel. "Quadratic Engel Curves and Consumer Demand." *Review of Economics and Statistics* 79(1997):527–539. doi: 10.1162/003465397557015.

Bray, G. A., S. J. Nielsen, and B. M. Popkin. "Consumption of High-Fructose Corn Syrup in Beverages May Play a Role in the Epidemic of Obesity." *American Journal of Clinical Nutrition* 79(2004):537–543.

Carpentier, A., and H. Guyomard. "Unconditional Elasticities in Two-Stage Demand Systems: An Approximate Solution." *American Journal of Agricultural Economics* 83(2001):222–229. doi: 10.1111/0002-9092.00149.

Chang, D., and A. Serletis. "Imposing Local Curvature in the QUAIDS." *Economics Letters* 115(2012):41–43. doi: 10.1016/j.econlet.2011.11.033.

Deaton, A., and J. Muellbauer. "An Almost Ideal Demand System." *American Economic Review* 70(1980):312–326.

Dhar, T., J.-P. Chavas, and B. W. Gould. "An Empirical Assessment of Endogeneity Issues in Demand Analysis for Differentiated Products." *American Journal of Agricultural Economics* 85(2003):605–617. doi: 10.1111/1467-8276.00459.

Dharmasena, S., and O. Capps. "Intended and Unintended Consequences of a Proposed National Tax on Sugar-Sweetened Beverages to Combat the U.S. Obesity Problem." *Health Economics* 21(2012):669–694. doi: 10.1002/hec.1738.

Globe Newswire. "Federal Judge Sets Stage for Full Trial as She Rejects Motions for Summary Judgements in Sugar Farmers False Advertising Lawsuit Against High Fructose Corn Syrup (HFCS) Manufacturers." 2015. Available online at http://globenewswire.com/news-release/2015/07/08/750723/10141085/en/Federal-Judge-Sets-Stage-for-full-Trial-as-She-Rejects-Motions-for-Summary-Judgements-in-Sugar-Farmers-False-Advertising-Lawsuit-Against-High-Fructose-Corn-Syrup-HFCS-Manufacturers.html.

Hall, B. H., and C. Cummins. *TSP 5.1 Reference Manual*. Palo Alto, CA: TSP International, 2009. Available online at http://tspintl.com/products/TSP51UG.pdf.

Hovhannisyan, V., and B. W. Gould. "Quantifying the Structure of Food Demand in China: An Econometric Approach." *Agricultural Economics* 42(2011):1–18. doi: 10.1111/j.1574-0862.2011.00548.x.

Johnson, R. K., L. J. Appel, M. Brands, B. V. Howard, M. Lefevre, R. H. Lustig, F. Sacks, L. M. Steffen, and J. Wylie-Rosett. "Dietary Sugars Intake and Cardiovascular Health: A Scientific Statement From the American Heart Association." *Circulation* 120(2009):1011–1020. doi: 10.1161/CIRCULATIONAHA.109.192627.

Lewis, K., and S. Basu. "Like Taking Candy From A Baby? The Not-So-Easy Policy Landscape For Cutting Added Sugars In America." 2015. Available online at http://healthaffairs.org/blog/2015/04/23/like-taking-candy-from-a-baby-the-not-so-easy-policy-landscape-for-cutting-added-sugars-in-america/.

Lipton, E. "Rival Industries Sweet-Talk the Public." *New York Times* (2014). Available online at http://www.nytimes.com/2014/02/12/business/rival-industries-sweet-talk-the-public.html.

Miao, Z., J. C. Beghin, and H. H. Jensen. "Taxing Sweets: Sweetener Input Tax or Final Consumption Tax." *Contemporary Economic Policy* 30(2012):344–361. doi: 10.1111/j.1465-7287.2011.00278.x.

Moschini, G., D. Moro, and R. D. Green. "Maintaining and Testing Separability in Demand Systems." *American Journal of Agricultural Economics* 76(1994):61–73. doi: 10.2307/1243921.

Moss, C. B., and A. Schmitz. "Trade in HFCS: Cointegration with Substitute Goods." In A. Schmitz, T. H. Spreen, W. A. Messina, and C. B. Moss, eds., *Sugar and Related Sweetener Markets: International Perspectives,* New York: CABI, 2002, 299–314.

Ryan, D. L., and T. J. Wales. "A Simple Method for Imposing Local Curvature in Some Flexible Consumer-Demand Systems." *Journal of Business & Economic Statistics* 16(1998):331–338. doi: 10.1080/07350015.1998.10524772.

Seale, J. L., M. A. Marchant, and A. Basso. "Imports versus Domestic Production: A Demand System Analysis of the U.S. Red Wine Market." *Review of Agricultural Economics* 25(2003):187–202. doi: 10.1111/1467-9353.00053.

Sellen, D., and E. Goddard. "Weak separability in coffee demand systems." *European Review of Agricultural Economics* 24(1997):133–144. doi: 10.1093/erae/24.1.133.

Slottje, D. J., ed. *Quantifying Consumer Preferences*, *Contributions to Economic Analysis*, vol. 288. Bingley, UK: Emerald Group Publishing, 2009.

U.S. Census Bureau. "Income." 2013. Available online at https://www.census.gov/topics/income-poverty/income.html.

U.S. Congress. "H.R.1687." 2015. Available online at https://www.congress.gov/114/bills/hr1687/BILLS-114hr1687ih.pdf.

U.S. Department of Agriculture. "Sugar and Sweetener Yearbook Tables. Sugar Briefing Room." 2014. Available online at http://www.ers.usda.gov/data-products/sugar-and-sweeteners-yearbook-tables.aspx.

U.S. Department of Agriculture, and U.S. Department of Health and Human Services. *Dietary Guidelines for Americans*. Washington, DC: U.S. Government Printing Office, 2010, 7th ed. Available online at http://www.health.gov/dietaryguidelines/dga2010/DietaryGuidelines2010.pdf.

U.S. Department of Agriculture, Economic Research Service. "Commodity and Food Elasticities." 2008. Available online at http://www.ers.usda.gov/data-products/commodity-and-food-elasticities.aspx.

Watson, E. "FDA Proposes Including DV for Added Sugars on the Nutrition Facts Panel." 2015. Available online at http://www.foodnavigator-usa.com/Regulation/FDA-proposes-including-DV-for-added-sugars-on-Nutrition-Facts-panel.

World Bank. "Population, Total." 2014. Available online at http://data.worldbank.org/indicator/SP.POP.TOTL.

Zhen, C., E. A. Finkelstein, J. M. Nonnemaker, S. A. Karns, and J. E. Todd. "Predicting the Effects of Sugar-Sweetened Beverage Taxes on Food and Beverage Demand in a Large Demand System." *American Journal of Agricultural Economics* 96(2014):1–25. doi: 10.1093/ajae/aat049.

Zhen, C., M. K. Wohlgenant, S. Karns, and P. Kaufman. "Habit Formation and Demand for Sugar-Sweetened Beverages." *American Journal of Agricultural Economics* 93(2011):175–193. doi: 10.1093/ajae/aaq155.

Appendix: Weak Separability Restrictions for QUAIDS

Elasticity of substitution (σ_{ik}) between good i and good k is given by the following equation (Moschini, Moro, and Green, 1994):

(A1) $$\sigma_{ik} = \frac{\varepsilon_{ik}^{H}}{w_k} = \frac{\varepsilon_{ik}^{M} + \epsilon_i \times w_k}{w_k} = \frac{\varepsilon_{ik}^{M}}{w_k} + \epsilon_i,$$

where ε_{ik}^{H} is Hicksian elasticity, ε_{ik}^{M} is Marshallian elasticity, ϵ_i is expenditure elasticity, and w_k is budget/expenditure share.

For the Quadratic AIDS model, when i is not equal to j, we have

(A2) $$\varepsilon_{ik}^{M} = \left[\frac{\gamma_{ik} - \left(\beta_i + \frac{2\lambda_i}{b(p)}\{\ln[\frac{m}{P}]\}\right)(\alpha_k + \sum_i \gamma_{ik} \ln p_i \ln p_k) - \frac{\lambda_i \beta_k}{b(p)}\{\ln[\frac{m}{P}]\}^2}{w_i}\right]$$

(A3) $$\epsilon_i = \frac{\beta_i + \frac{2\lambda_i}{b(p)}\{\ln[\frac{m}{P})]\}}{w_i} + 1 = \frac{\beta_i + \frac{2\lambda_i}{b(p)}\{\ln[\frac{m}{P}]\} + w_i}{w_i}.$$

Substituting ϵ_{ij}^{M} and ϵ_i in the elasticity of substitution equation (A1) gives

(A4)
$$\sigma_{ik} = \left[\frac{\gamma_{ik} - \left(\beta_i + \frac{2\lambda_i}{b(p)}\{\ln[\frac{m}{P}]\}\right)(\alpha_k + \sum_i \gamma_{ik} \ln p_i \ln p_k) - \frac{\lambda_i \beta_k}{b(p)}\{\ln[\frac{m}{P}]\}^2 + \left(\beta_i + \frac{2\lambda_i}{b(p)}\{\ln[\frac{m}{P}]\}\right) \times w_k + w_i w_k}{w_i w_k}\right].$$

According to Moschini, Moro, and Green (1994),

(A5) $$\frac{\sigma_{ik}}{\sigma_{jm}} = \frac{\epsilon_i \epsilon_k}{\epsilon_j \epsilon_m},$$

where i and j belong to Ig group and k and m belong to Is group.

But, in our case, (i,j) belong to one group and only k is in another group. Therefore, $k = m$. Hence, the above equation transformed as

(A6) $$\frac{\sigma_{ik}}{\sigma_{jk}} = \frac{\epsilon_i \epsilon_k}{\epsilon_j \epsilon_k} = \frac{\epsilon_i}{\epsilon_j}.$$

Substituting the terms σ_{ik}, σ_{jm}, ϵ_i, and ϵ_j into equation (A6) gives

(A7)
$$\frac{\gamma_{ik} - \left(\beta_i + \frac{2\lambda_i}{b(p)}\{\ln[\frac{m}{P}]\}\right)(\alpha_k + \sum_i \gamma_{ik} \ln p_i \ln p_k) - \frac{\lambda_i \beta_k}{b(p)}\{\ln[\frac{m}{P}]\}^2 + \left(\beta_i + \frac{2\lambda_i}{b(p)}\{\ln[\frac{m}{P}]\}\right) \times w_k + w_i w_k}{\gamma_{jk} - \left(\beta_j + \frac{2\lambda_j}{b(p)}\{\ln[\frac{m}{P}]\}\right)(\alpha_k + \sum_j \gamma_{jk} \ln p_j \ln p_k) - \frac{\lambda_j \beta_k}{b(p)}\{\ln[\frac{m}{P}]\}^2 + \left(\beta_j + \frac{2\lambda_j}{b(p)}\{\ln[\frac{m}{P}]\}\right) \times w_k + w_j w_k} = \frac{\left(\beta_i + \frac{2\lambda_i}{b(p)}\{\ln[\frac{m}{P}]\} + w_i\right)}{\left(\beta_j + \frac{2\lambda_j}{b(p)}\{\ln[\frac{m}{P}]\} + w_j\right)}$$

(A8)
$$\frac{\gamma_{ik} - \left(\beta_i + \frac{2\lambda_i}{b(p)}\{\ln[\frac{m}{P}]\}\right)[(\alpha_k + \sum_i \gamma_{ik} \ln p_i \ln p_k) - w_k] - \frac{\lambda_i \beta_k}{b(p)}\{\ln[\frac{m}{P}]\}^2 + w_i w_k}{\gamma_{jk} - \left(\beta_j + \frac{2\lambda_i}{b(p)}\{\ln[\frac{m}{P}]\}\right)[(\alpha_k + \sum_j \gamma_{jk} \ln p_j \ln p_k) - w_k] - \frac{\lambda_i \beta_k}{b(p)}\{\ln[\frac{m}{P}]\}^2 + w_j w_k} = \frac{\left(\beta_i + \frac{2\lambda_i}{b(p)}\{\ln[\frac{m}{P}]\} + w_i\right)}{\left(\beta_j + \frac{2\lambda_i}{b(p)}\{\ln[\frac{m}{P}]\} + w_j\right)}$$

At a point where price and expenditure are assumed as unity, we have $b(p) = 1$, $\ln\left[\frac{m}{P}\right] = -\alpha_0$, and $w_i = \alpha_i - \beta_i \alpha_0 + \lambda_i \alpha_0^2$. Substituting these terms in the above equations (A7)–(A8) gives the following separability restriction:

$$\frac{\gamma_{ik}-\alpha_0(\beta_i\beta_k+\alpha_i\beta_k+\alpha_k\beta_i)+\alpha_0^2(\beta_i\lambda_k+\lambda_i\beta_k+\lambda_k\alpha_i+\beta_i\beta_k+\lambda_i\alpha_k)-\alpha_0^3(2\lambda_i\lambda_k+\lambda_k\beta_i+\lambda_i\beta_k)+\alpha_0^4\lambda_i\lambda_k+\alpha_i\alpha_k}{\gamma_{jk}-\alpha_0(\beta_j\beta_k+\alpha_j\beta_k+\alpha_k\beta_j)+\alpha_0^2(\beta_j\lambda_k+\lambda_j\beta_k+\lambda_k\alpha_j+\beta_j\beta_k+\lambda_j\alpha_k)-\alpha_0^3(2\lambda_j\lambda_k+\lambda_k\beta_j+\lambda_j\beta_k)+\alpha_0^4\lambda_j\lambda_k+\alpha_j\alpha_k} =$$

$$\frac{\beta_i+\alpha_i+\alpha_0(-2\lambda_i-\beta_i+\lambda_i\alpha_0)}{\beta_j+\alpha_j+\alpha_0(-2\lambda_j-\beta_j+\lambda_j\alpha_0)} \tag{A9}$$

The above separability restriction is nonhomothetic and it is imposed on the Quadratic AIDS estimation.